MATH 140A – MOCK FINAL EXAM

1. *(10 points)* Suppose S is a nonempty and bounded subset of $\mathbb{R}$. Show that

$$\inf(S) = -\sup(-S)$$

Date: Tuesday, June 9, 2020.

2. *(10 points)*

Suppose (s_n) and (t_n) are convergent sequences in $\mathbb{R}$ with $s_n \to s$ and $t_n \to t$. Show using the definition of a limit that

$$\lim_{n\to\infty} s_n - t_n = s - t$$

3. *(10 = 8+2 points)*

(a) Show that if (s_n) is a bounded sequence in $\mathbb{R}$ and $k \geq 0$, then

$$\limsup_{n\to\infty} ks_n = k\left(\limsup_{n\to\infty} s_n\right)$$

(b) Is the same result true if you replace $k \geq 0$ by $k \in \mathbb{R}$?

4. *(10 = 4 + 4 + 2 points)* Let (S, d) be a metric space

(a) Show that the union of any number of open subsets of S is open

(b) Show that the intersection of finitely many open subsets of S is open

(c) Is the intersection of infinitely many open subsets of S always open?

Suggestion: I *highly* recommend also checking out the metric space problems on Mock Midterm 2.

5. (10 points) Prove the integral test in the case of

$$\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$$

6. *(10 points)* Suppose f is a continuous function on $(-1,1)$ that satisfies $f(x^2) = f(x)$ for all x. Show that f is constant

Hint: Show that $f(x) = f(0)$ for all x. For this, calculate $f(x^4)$, $f(x^8)$, and, more generally, $f(x^{2^n})$ for $n \geq 0$

7. *(10 points)* Suppose $y \geq 0$ is given. Show that there is a unique $x \geq 0$ such that $y = x^2$

8. *(10 points)* Show directly, using the definition of uniform continuity that $f(x) = x^2$ is not uniformly continuous on $[0, \infty)$

Hint: This is similar to (but easier than) Example 3 in Lecture 27. Let $\epsilon > 0$ be TBA and fix δ. Let $a = |x - y|$ and assume $x < y$.