MOCK FINAL

PEYAM RYAN TABRIZIAN

Instructions: This is a mock final, designed to give you some practice for the actual final. Do **NOT** expect the questions on the final to be the same; some will be easier, but most will be harder. Please also look at the study guide and the suggested homework for a more complete study experience!

1		10
2		10
3		10
4		10
5		10
6		10
7		10
8		10
9		10
10		10
Total		100

Date: Wednesday, June 13, 2018.

1. *(10 points)* Use normal vectors to find the line of intersection of the planes $x + 2y + 3z = 1$ and $x - y + z = 1$.

2. *(10 points)* Is the following function continuous at $(0,0)$?

$$f(x,y) = \begin{cases} \dfrac{xy}{x^2+xy+y^2} & \text{if } (x,y) \neq (0,0) \\ 0 & \text{if } (x,y) = (0,0) \end{cases}$$

3. *(10 points)* Find an approximate value of

$$\sqrt{(4.2)^2 + (0.1)^2 + (2.9)^2}$$

4. *(10 points)* Find $\frac{\partial z}{\partial x}$ at $(0, 1)$ where $\ln(z) = xyz$

5. *(10 points)* Show that the equation of the tangent plane to the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ at the point (x_0, y_0, z_0) can be written as:

$$\frac{xx_0}{a^2} + \frac{yy_0}{b^2} + \frac{zz_0}{c^2} = 1$$

6. *(10 points)* Find the local maximum and minimum values and saddle points of the function $f(x, y) = x^4 - 2x^2 + y^3 - 3y$.

7. *(10 points)* **Note:** This question has two parts to give you extra practice, but a more reasonable question about this on the final would only have one part.

   (a) Use Lagrange multipliers to show that among all boxes with fixed volume V, the one with the smallest surface area must be a cube.

(b) Use Lagrange multipliers to show that amoung all boxes with fixed surface area S, the one with the largest volume must be a cube.

8. *(10 points)* Calculate

$$\int_0^1 \int_{x^2}^1 \sqrt{y}\sin(y)dydx$$

9. *(10 points)* Find the volume of the solid below the function $z = \sqrt{x^2 + y^2}$ and above the ring $1 \leq x^2 + y^2 \leq 4$

10. *(10 points)* Calculate

$$\int\int\int_E z \, dxdydz$$

where E is the solid in the first octant bounded by the cylinder $y^2 + z^2 = 9$ and the planes $x = 0$, $y = 3x$, and $z = 0$.