MATH 140A – MOCK MIDTERM 2

Note: This mock midterm has 5 questions, in order to give you extra practice. On the actual midterm, there will be only 4 questions.

1. *(10 points)* Suppose $s \in \mathbb{R}$ and let (s_n) be a sequence in $\mathbb{R}$ such that for every $r > 0$, $(s-r, s+r)$ contains infinitely many terms of (s_n). Use an inductive construction to find a subsequence (s_{n_k}) of (s_n) that converges to s

Date: Monday, May 18, 2020.

2. *(10 points)* Show directly, without using the ratio or root test, that if (a_n) is a sequence with $\limsup_{n\to\infty} |a_n|^{\frac{1}{n}} = \alpha < 1$, then $\sum a_n$ converges absolutely.

Note: Start by letting $\epsilon > 0$ such that $\alpha + \epsilon < 1$

3. *(10 points)* Show directly that $(\mathbb{R}^2, d_1)$ is complete, where .

$$d_1\left((x_1, x_2), (y_1, y_2)\right) = |x_1 - y_1| + |x_2 - y_2|$$

Note: You may only use the fact that $\mathbb{R}$ (with its usual metric) is complete.

4. (10 points) Let (S, d) be any metric space and $E \subseteq S$. Show:

$$\overline{E^c} = (E^\circ)^c$$

Note: Here (and on the exam), please use the following definitions of interior and closure (here F is any subset of S)

Definition:

$x \in F^\circ$ if there is $r > 0$ such that $B(x, r) \subseteq F$
$s \in \overline{F}$ if there is a sequence (s_n) in F that converges to s

5. *(10 = 7 + 3 points)* Let (S, d) be any metric space

(a) Show that the union of any two compact subsets of S is always compact

(b) Is the intersection of any two compact subsets of S always compact? Why or why not?