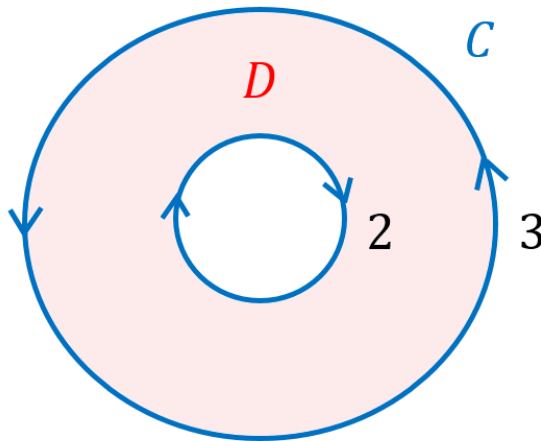


MATH 251 – QUIZ 10 – SOLUTIONS

Question 1: (5 points)

STEP 1: Picture:



Note: Note the clockwise orientation of the inner circle. This is because if you walk along that inner curve, D has to be to your left.

STEP 2: Inequalities

$$\begin{cases} 2 \leq r \leq 3 \\ 0 \leq \theta \leq 2\pi \end{cases}$$

STEP 3: Integrate

Date: Friday, December 3, 2021.

$$\begin{aligned} & \int_C F \cdot dr \\ &= \int \int_D \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right) dx dy \\ &= \int \int_D (4x^3 + y^2)_x - (-4y^3 + x^2)_y dx dy \\ &= \int \int_D 12x^2 - (-12y^2) dx dy \\ &= \int \int_D 12(x^2 + y^2) dx dy \\ &= \int_0^{2\pi} \int_2^3 12r^2 r dr d\theta \\ &= 2\pi \int_2^3 12r^3 dr \\ &= 2\pi [3r^4]_2^3 \\ &= 6\pi (3^4 - 2^4) \\ &= 6\pi(81 - 16) \\ &= 6\pi(65) \\ &= 390\pi \end{aligned}$$

Question 2: (5 points)

STEP 1: Slopes

$$\begin{aligned} r_u &= \langle \cos(v), \sin(v), 1 \rangle \\ r_v &= \langle -u \sin(v), u \cos(v), 0 \rangle \end{aligned}$$

STEP 2: Normal Vector

$$\begin{aligned}
\hat{n} &= r_u \times r_v \\
&= \begin{vmatrix} i & j & k \\ \cos(v) & \sin(v) & 1 \\ -u \sin(v) & u \cos(v) & 0 \end{vmatrix} \\
&= \langle -u \cos(v), -u \sin(v), u \cos^2(v) + u \sin^2(v) \rangle \\
&= \langle -u \cos(v), -u \sin(v), u \rangle
\end{aligned}$$

STEP 3: dS

$$dS = \|r_u \times r_v\| = \sqrt{u^2 \cos^2(v) + u^2 \sin^2(v) + u^2} = \sqrt{u^2 + u^2} = (\sqrt{2}) u$$

STEP 4: Integrate

$$\begin{aligned}
&\iint_S dS \\
&= \iint_D (\sqrt{2}) u \, du \, dv \\
&= \sqrt{2} \int_0^\pi \int_0^3 u \, du \, dv \\
&= (\sqrt{2}) \pi \int_0^3 u \, du \\
&= (\sqrt{2}) \pi \left[\frac{u^2}{2} \right]_0^3 \\
&= (\sqrt{2}) \pi \left(\frac{9}{2} \right) \\
&= \left(\frac{9\sqrt{2}}{2} \right) \pi
\end{aligned}$$