MATH 251 – QUIZ 2 – SOLUTIONS

Question 1:

L_1 has direction vector $\langle 1, -2, -3 \rangle$ and L_2 has direction vector $\langle 1, 3, -7 \rangle$, which are not parallel, and hence the lines are not parallel.

To find out if they're skew or intersecting, solve both equations at the same time:

$$\begin{cases} 2 + t = 3 + s \\ 3 - 2t = -4 + 3s \\ 1 - 3t = 2 - 7s \end{cases}$$

From the first equation $2 + t = 3 + s$, we get $t = 3 + s - 2$ so $t = 1 + s$

Then the second equation becomes:

$$\begin{aligned} 3 - 2t &= -4 + 3s \\ 3 - 2(1 + s) &= -4 + 3s \\ 3 - 2 - 2s &= -4 + 3s \\ 1 - 2s &= -4 + 3s \\ 3s + 2s &= 1 + 4 \\ 5s &= 5 \\ s &= 1 \end{aligned}$$

And therefore $t = 1 + s = 1 + 1 = 2$.

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Hence we get $t = 2$ and $s = 1$.

And the third equation then becomes:

$$\begin{aligned} 1 - 3t &\stackrel{?}{=} 2 - 7s \\ 1 - 3(2) &\stackrel{?}{=} 2 - 7(1) \\ -5 &\stackrel{?}{=} -5 \checkmark \end{aligned}$$

Hence the lines are **intersecting**, and to find the point of intersection you let $t = 2$ in L_1:

$$\begin{cases} x(2) = 2 + 2 = 4 \\ y(2) = 3 - 2(2) = -1 \\ z(2) = 1 - 3(2) = -5 \end{cases}$$

Hence the point of intersection is $\boxed{(4, -1, -5)}$

Optional Check: To check your answer, let $s = 1$ in the equation for L_2:

$$\begin{cases} x(1) = 3 + 1 = 4 \\ y(1) = -4 + 3(1) = -1 \\ z(1) = 2 - 7(1) = -5 \end{cases}$$

Question 2:

Notice that the plane contains three points: $A = (1, 0, 4)$, $B = \mathbf{r}(0) = \langle 3, 5, 2 \rangle$ and $C = \mathbf{r}(1) = \langle 7, 4, 3 \rangle$, so all we need to do is find the equation of the plane going through three points.

(1) **Point:** $A = (1, 0, 4)$

(2) **Normal Vector**

(i) The following vectors **a** and **b** are on the plane:

$$\begin{aligned}\mathbf{a} =& \overrightarrow{AB} = \langle 3-1, 5-0, 2-4\rangle = \langle 2, 5, -2\rangle \\ \mathbf{b} =& \overrightarrow{AC} = \langle 7-1, 4-0, 3-4\rangle = \langle 6, 4, -1\rangle\end{aligned}$$

(ii)

$$\begin{aligned}\mathbf{n} =& \mathbf{a}\times\mathbf{b} \\ =& \begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & 5 & -2 \\ 6 & 4 & -1\end{vmatrix} \\ =& \begin{vmatrix}5 & -2 \\ 4 & -1\end{vmatrix}\mathbf{i} - \begin{vmatrix}2 & -2 \\ 6 & -1\end{vmatrix}\mathbf{j} + \begin{vmatrix}2 & 5 \\ 6 & 4\end{vmatrix}\mathbf{k} \\ =& (-5+8)\mathbf{i} - (-2+12)\mathbf{j} + (8-30)\mathbf{k} \\ =& \langle 3, -10, -22\rangle\end{aligned}$$

So $\mathbf{n} = \langle 3, -10, -22\rangle$

(3) **Equation:** (Recall the point was $A = (1, 0, 4)$)

$$3(x-1) - 10y - 22(z-4) = 0$$