

## MATH 251 – QUIZ 4 – SOLUTIONS

### Question 1:

Let  $f(x, y) = \sqrt{x^2 + 9e^y}$  and our point be  $(4, 0)$

#### Option 1: Linear Approximations:

$$f(4, 0) = \sqrt{4^2 + 9} = \sqrt{25} = 5$$

$$f_x(x, y) = \frac{x}{\sqrt{x^2 + 9e^y}} \Rightarrow f_x(4, 0) = \frac{4}{\sqrt{16 + 9}} = \frac{4}{5}$$

$$f_y(x, y) = \frac{9e^y}{2\sqrt{x^2 + 9e^y}} \Rightarrow f_y(4, 0) = \frac{9}{2\sqrt{4^2 + 9}} = \frac{9}{2 \times 5} = \frac{9}{10}$$

$$L(x, y) = f(4, 0) + f_x(4, 0)(x - 4) + f_y(4, 0)(y - 0) = 5 + \frac{4}{5}(x - 4) + \frac{9}{10}(y - 0)$$

$$\begin{aligned}\sqrt{(4.01)^2 + 9e^{-0.02}} &= f(4.01, -0.02) \\ &\approx L(4.01, -0.02) \\ &= 5 + \frac{4}{5}(4.01 - 4) + \frac{9}{10}(-0.02 - 0) \\ &= 5 + (0.8)(0.01) + (0.9)(-0.02) \\ &= 5 + 0.008 - 0.018 \\ &= 5 - 0.01 \\ &= 4.99\end{aligned}$$

#### Differentials:

$$dx = \Delta x = 4.01 - 4 = 0.01$$

$$dy = \Delta y = 0 - 0.02 = -0.02$$

$$\begin{aligned}dz &= f_x(4, 0)dx + f_y(4, 0)dy \\ &= \frac{4}{5}(0.01) + \frac{9}{10}(-0.02) \\ &= 0.8(0.01) + 0.9(-0.02) \\ &= 0.008 - 0.018 \\ &= -0.01\end{aligned}$$

$$\begin{aligned}\Delta z &\approx dz \\ f(4.01, -0.02) - f(4, 0) &\approx -0.01 \\ \sqrt{(4.01)^2 + 9e^{-0.02}} - 5 &\approx -0.01 \\ \sqrt{(4.01)^2 + 9e^{-0.02}} &\approx 5 - 0.01 = 4.99\end{aligned}$$

**Question 2:**

By the Chain Rule:

$$\frac{\partial z}{\partial s} = \left(\frac{\partial z}{\partial x}\right) \left(\frac{\partial x}{\partial s}\right) + \left(\frac{\partial z}{\partial y}\right) \left(\frac{\partial y}{\partial s}\right) = (2x + y^2)(t) + (2xy)(2s)$$

At  $s = 1$  and  $t = -2$ , we have  $x = (1)(-2) = -2$  and  $y = (1)^2 + 1 = 2$ , and therefore

$$\frac{\partial z}{\partial s} = (2(-2) + 2^2)(-2) + 2(-2)(2)(2)(1) = 0 - 16 = -16$$