MATH 251 – QUIZ 5 – SOLUTIONS

Question 1:

(1) $F(x, y, z) = (x + y + z)^2 - (x^2 + y^2 + z^2)$

(2) **Point:** $(2, 2, -1)$

(3) **Normal Vector:**

$$\begin{aligned} F_x =& 2(x + y + z) - 2x \\ F_y =& 2(x + y + z) - 2y \\ F_z =& 2(x + y + z) - 2z \end{aligned}$$

$\nabla F = \langle F_x, F_y, F_z \rangle = \langle 2(x + y + z) - 2x, 2(x + y + z) - 2y, 2(x + y + z) - 2z \rangle$

$$\begin{aligned} \mathbf{n} =& \nabla F(2, 2 - 1) \\ =& \langle 2(2 + 2 - 1) - 2(2), 2(2 + 2 - 1) - 2(2), 2(2 + 2 - 1) - 2(-1) \rangle \\ =& \langle 2(3) - 4, 2(3) - 4, 2(3) + 2 \rangle \\ =& \langle 2, 2, 8 \rangle \end{aligned}$$

(4) Therefore, the equation of the tangent plane is

$$2(x - 2) + 2(y - 2) + 8(z + 1) = 0$$

Question 2:

STEP 1: Critical Points

$$\begin{cases} f_x =& 4x^3 - 4x = 4x(x^2 - 1) = 0 \Rightarrow x = 0 \text{ or } x^2 = 1 \Rightarrow x = 0 \text{ or } x = \pm 1 \\ f_y =& 3y^2 - 3 = 0 \Rightarrow y^2 = 1 \Rightarrow y = \pm 1 \end{cases}$$

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This gives us 6 critical points: $(0, \pm 1), (1, \pm 1), (-1, \pm 1)$

STEP 2: Second derivatives

$$D(x,y) = \begin{vmatrix} f_{xx} & f_{xy} \\ f_{yx} & f_{yy} \end{vmatrix} = \begin{vmatrix} 12x^2 - 4 & 0 \\ 0 & 6y \end{vmatrix}$$

$$D(0,1) = \begin{vmatrix} -4 & 0 \\ 0 & 6 \end{vmatrix} = -24 < 0$$

Hence f has a saddle point at $(0,1)$

$$D(0,-1) = \begin{vmatrix} -4 & 0 \\ 0 & -6 \end{vmatrix} = 24 > 0 \text{ and } f_{xx}(0,-1) = -4 < 0$$

Hence f has a local max at $(0,-1)$

$$D(1,1) = \begin{vmatrix} 8 & 0 \\ 0 & 6 \end{vmatrix} = 48 > 0 \text{ and } f_{xx}(1,1) = 8 > 0$$

Hence f has a local min at $(1,1)$

$$D(1,-1) = \begin{vmatrix} 8 & 0 \\ 0 & -6 \end{vmatrix} = -48 < 0$$

Hence f has a saddle point at $(1,-1)$

$$D(-1,1) = \begin{vmatrix} 8 & 0 \\ 0 & 6 \end{vmatrix} = 48 > 0 \text{ and } f_{xx}(-1,1) = 8 > 0$$

Hence f has a local min at $(-1,1)$

$$D(-1,-1) = \begin{vmatrix} 8 & 0 \\ 0 & -6 \end{vmatrix} = -48$$

Hence f has a saddle point at $(-1,-1)$

STEP 3: Conclusion

Saddle Points at:	$(0,1),(1,-1),(-1,-1)$
Local Max at:	$(0,-1)$
Local Min at:	$(1,1),(-1,1)$