MATH 251 – QUIZ 6 – SOLUTIONS

Question 1:

STEP 1: Picture
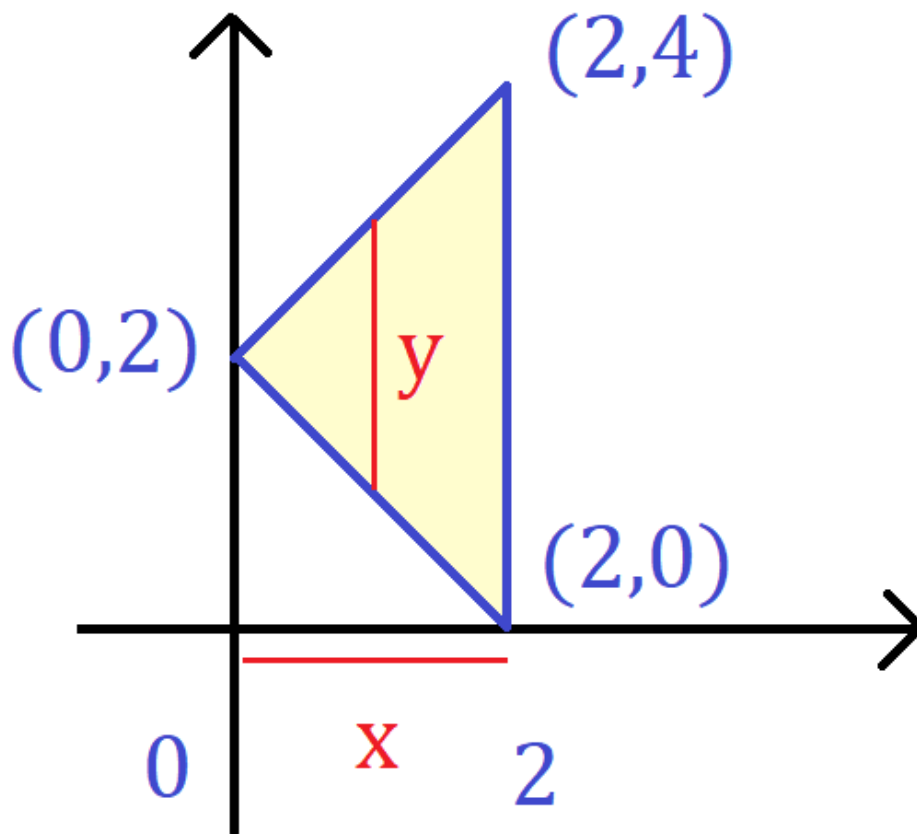
STEP 2: Inequalities:

Here the smaller function is $y = 2 - x$ (the line connecting $(0, 2)$ and $(2, 0)$) and the smaller bigger is $y = x + 2$ (the line connecting $(0, 2)$ and $(2, 4)$), therefore

$$2 - x \leq y \leq x + 2$$
$$0 \leq x \leq 2$$

STEP 3: Integrate

$$\int \int_D 2xy\,dx\,dy$$
$$= \int_0^2 \int_{2-x}^{x+2} 2xy\,dy\,dx$$
$$= \int_0^2 \left[xy^2\right]_{y=2-x}^{y=x+2} dx$$
$$= \int_0^2 x\left(x + 2\right)^2 - x\left(2 - x\right)^2 dx$$
$$= \int_0^2 x\left(\left(x + 2\right)^2 - \left(2 - x\right)^2\right) dx$$
$$= \int_0^2 x\left(x^2 + 4x + 4 - 4 + 4x - x^2\right) dx$$
$$= \int_0^2 x(8x)dx$$
$$= \int_0^2 8x^2 dx$$
$$= \left[\frac{8}{3}x^3\right]_0^2$$

$$= \frac{8}{3}(2)^3 - 0$$
$$= \frac{64}{3}$$

Question 2:

STEP 1: Picture

Here the region is

$$\sqrt{x} \leq y \leq 3$$
$$0 \leq x \leq 9$$
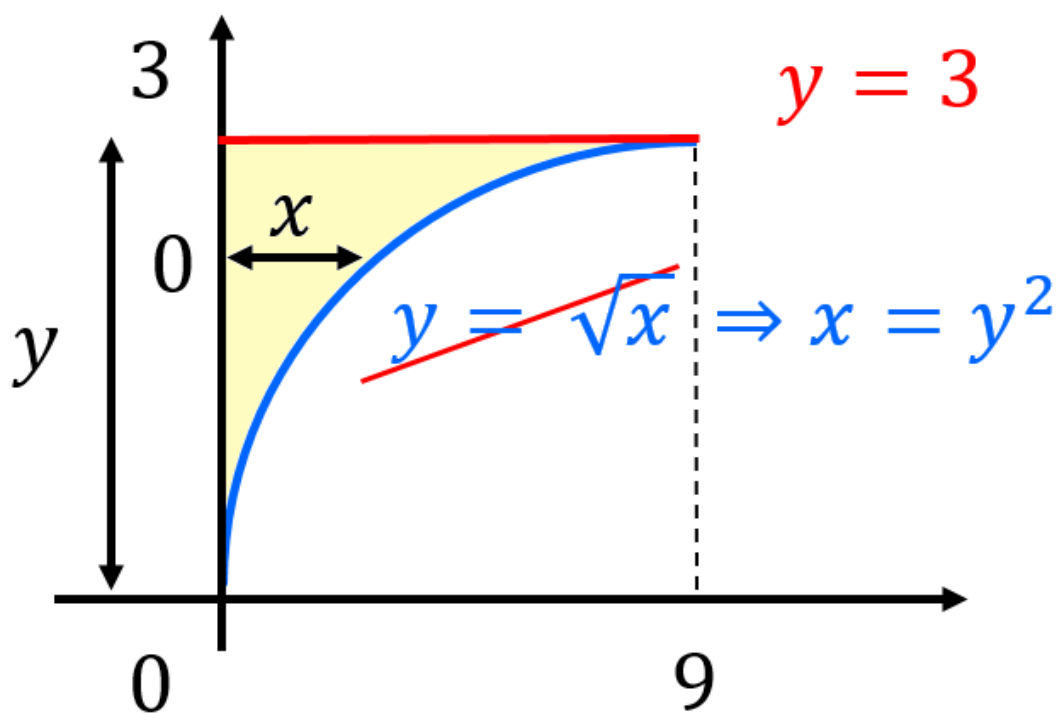
STEP 2: Inequalities

Notice $y = \sqrt{x} \Rightarrow x = y^2$, so writing this as a horizontal region, we get

$$0 \leq x \leq y^2$$
$$0 \leq y \leq 3$$

STEP 3: Integrate

$$\int_0^9 \int_{\sqrt{x}}^3 \frac{1}{y^3 + 1} dy dx = \int_0^3 \int_0^{y^2} \frac{1}{y^3 + 1} dx dy$$

$$= \int_0^3 \left(\frac{1}{y^3 + 1} \right) \left(y^2 - 0 \right) dy$$

$$= \int_0^3 \frac{y^2}{y^3 + 1} dy$$

$$= \left[\frac{1}{3} \ln \left| y^3 + 1 \right| \right]_0^3$$

$$= \frac{1}{3} \ln \left(27 + 1 \right) - \frac{1}{3} \ln \left(1 \right)$$

$$= \frac{1}{3} \ln \left(28 \right)$$