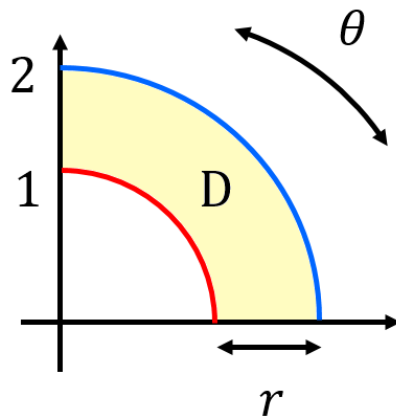


MATH 251 – QUIZ 7 – SOLUTIONS

Question 1:

STEP 1: Picture



STEP 2: Inequalities

$$\begin{cases} 1 \leq r \leq 2 \\ 0 \leq \theta \leq \frac{\pi}{2} \end{cases}$$

STEP 3: Integrate

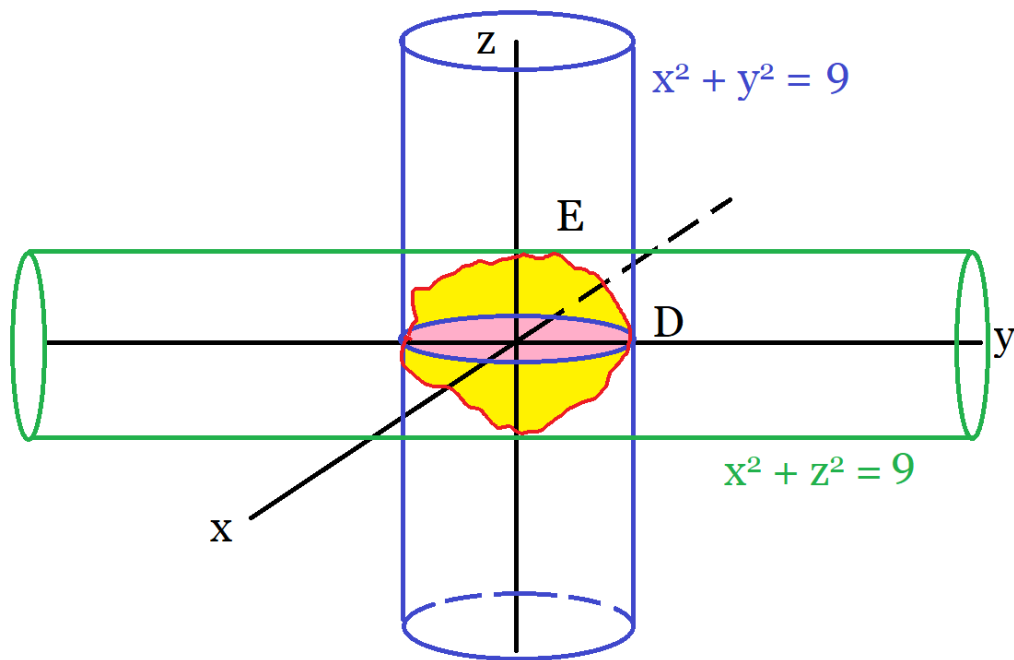
Date: Friday, October 29, 2021.

$$\begin{aligned}
\iint_D y^2 dx dy &= \int_0^{\frac{\pi}{2}} \int_1^2 (r \sin(\theta))^2 r dr d\theta \\
&= \int_0^{\frac{\pi}{2}} \int_1^2 r^3 \sin^2(\theta) dr d\theta \\
&= \int_1^2 r^3 dr \int_0^{\frac{\pi}{2}} \sin^2(\theta) d\theta \\
&= \left[\frac{r^4}{4} \right]_1^2 \int_0^{\frac{\pi}{2}} \frac{1}{2} - \frac{1}{2} \cos(2\theta) d\theta \\
&= \frac{1}{4} (2^4 - 1^4) \left[\frac{\theta}{2} - \frac{\sin(2\theta)}{4} \right]_0^{\frac{\pi}{2}} \\
&= \frac{15}{4} \left(\frac{\pi}{4} - 0 - \frac{1}{4} \sin(\pi) + \frac{1}{4} \sin(0) \right) \\
&= \frac{15}{4} \left(\frac{\pi}{4} \right) \\
&= \frac{15\pi}{16}
\end{aligned}$$

Question 2:

STEP 1: Picture:

$x^2 + y^2 = 9$ is a cylinder in the z -direction, and $x^2 + z^2 = 9$ is a cylinder in the y -direction.

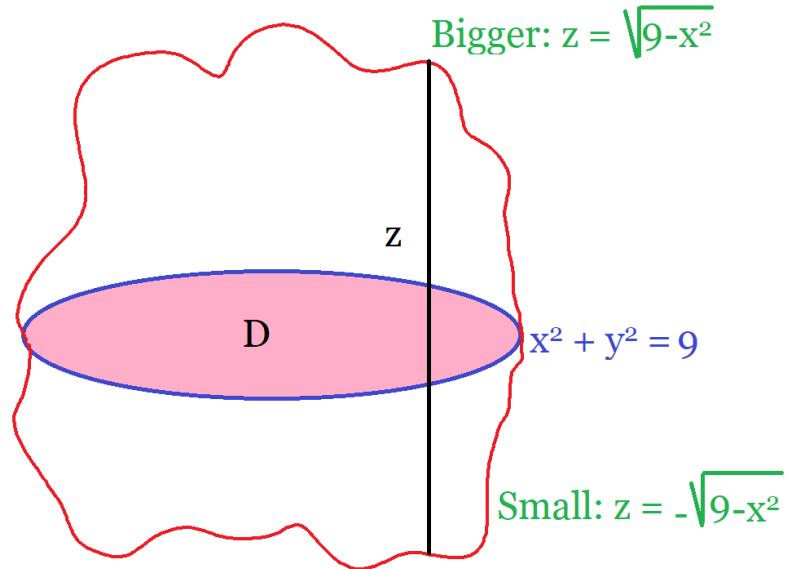


STEP 2: Inequalities:

Small $\leq z \leq$ Big

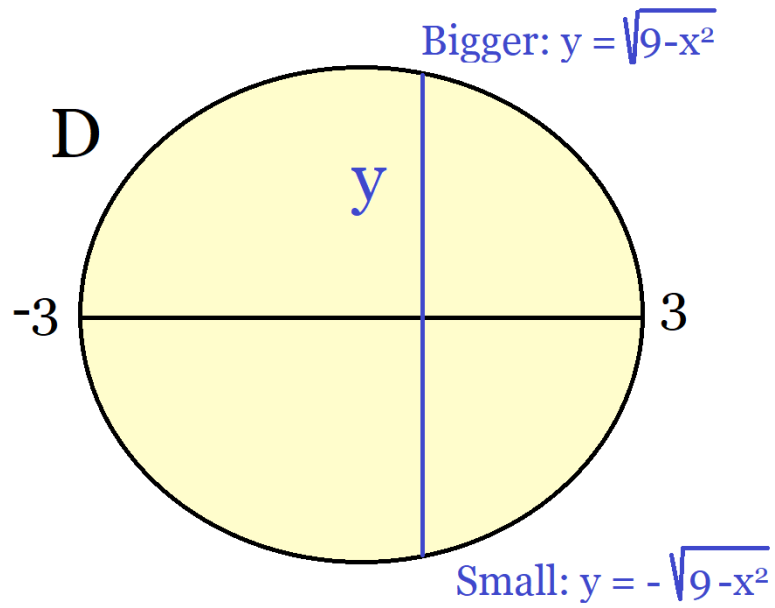
$$z^2 + x^2 = 9 \Rightarrow z^2 = 9 - x^2 \Rightarrow z = \pm\sqrt{9 - x^2}$$

$$-\sqrt{9 - x^2} \leq z \leq \sqrt{9 - x^2}$$



STEP 3: Find D

Using $x^2 + y^2 = 9$, D is a disk of radius 3



STEP 4:Small $\leq y \leq$ Big

$$x^2 + y^2 = 9 \Rightarrow y^2 = 9 - x^2 \Rightarrow -\sqrt{9 - x^2} \leq y \leq \sqrt{9 - x^2}$$

$$\begin{cases} -\sqrt{9 - x^2} \leq z \leq \sqrt{9 - x^2} \\ -\sqrt{9 - x^2} \leq y \leq \sqrt{9 - x^2} \\ -3 \leq x \leq 3 \end{cases}$$

STEP 5: Integrate:

$$\begin{aligned} \text{Vol}(E) &= \int \int \int_E 1 \, dx \, dy \, dz \\ &= \int_{-3}^3 \int_{-\sqrt{9-x^2}}^{\sqrt{9-x^2}} \int_{-\sqrt{9-x^2}}^{\sqrt{9-x^2}} dz \, dy \, dx \\ &= \int_{-3}^3 \int_{-\sqrt{9-x^2}}^{\sqrt{9-x^2}} \sqrt{9-x^2} - \left(-\sqrt{9-x^2}\right) \, dy \, dx \\ &= \int_{-3}^3 \int_{-\sqrt{9-x^2}}^{\sqrt{9-x^2}} 2\sqrt{9-x^2} \, dy \, dx \\ &= \int_{-3}^3 2\sqrt{9-x^2} \left(\sqrt{9-x^2} - \left(-\sqrt{9-x^2}\right)\right) \, dx \\ &= \int_{-3}^3 \left(2\sqrt{9-x^2}\right) \left(2\sqrt{9-x^2}\right) \, dx \end{aligned}$$

$$\begin{aligned} &= \int_{-3}^3 4(9 - x^2) dx \\ &= 4 \int_{-3}^3 (9 - x^2) dx \\ &= 4(2) \int_0^3 9 - x^2 dx && \text{(Since } 9 - x^2 \text{ is even)} \\ &= 8 \left[9x - \frac{1}{3}x^3 \right]_0^3 \\ &= 8 \left(9(3) - \frac{1}{3}27 \right) \\ &= 8(27 - 9) \\ &= 8(18) \\ &= 144 \end{aligned}$$