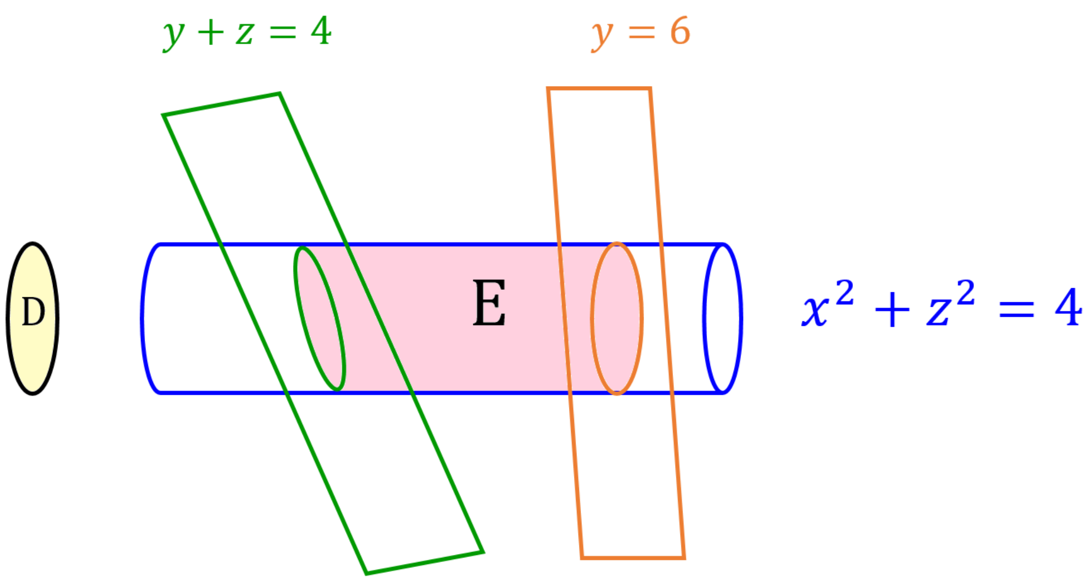
MATH 251 – QUIZ 8 – SOLUTIONS

Question 1: *(5 points)*

STEP 1: Picture

$x^2 + z^2 = 4$ is a cylinder in the $y-$direction, so the solid faces the y-direction



STEP 2: Inequalities

$y + z = 4 \Rightarrow y = 4 - z$

Date: Friday, November 5, 2021.

$$\begin{aligned} \text{Left } &\leq y \leq \text{ Right} \\ 4 - z &\leq y \leq 6 \\ 4 - r\sin(\theta) &\leq y \leq 6 \end{aligned}$$

STEP 3: Find D

D is a disk of radius 2, so we have

$$\begin{aligned} 0 \leq r \leq 2 \\ 0 \leq \theta \leq 2\pi \end{aligned}$$

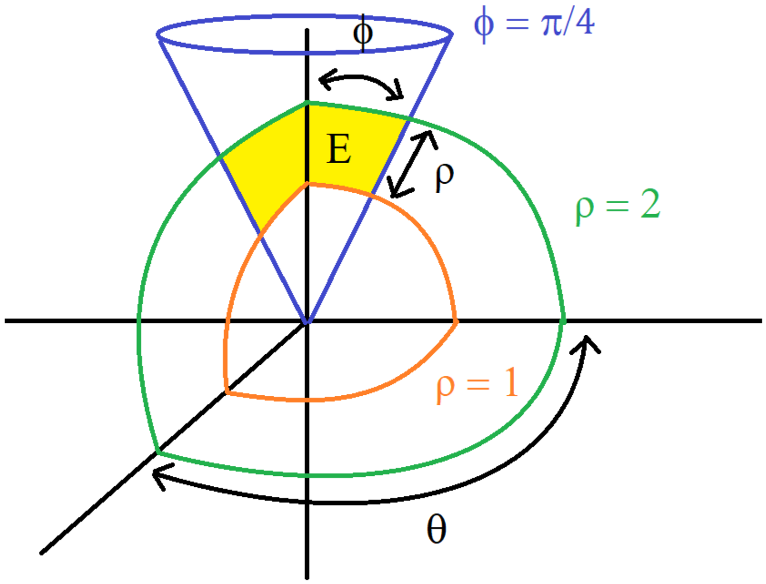
STEP 4: Integrate

$$\begin{aligned} &\int\int\int_E 4\, dxdydz \\ =&\int_0^{2\pi}\int_0^2\int_{4-r\sin(\theta)}^6 4r\; dydrd\theta \\ =&\int_0^{2\pi}\int_0^2 4r\left(6-4+r\sin(\theta)\right)\, drd\theta \\ =&\int_0^{2\pi}\int_0^2 8r+4r^2\sin(\theta)drd\theta \\ =&\int_0^{2\pi}\left[4r^2+\frac{4}{3}r^3\sin(\theta)\right]_{r=0}^{r=2} d\theta \\ =&\int_0^{2\pi} 16+\frac{4(8)}{3}\sin(\theta)d\theta \\ =&\left[16\theta-\frac{32}{3}\cos(\theta)\right]_0^{2\pi} \\ =&16(2\pi)-0 \\ =&32\pi \end{aligned}$$

Question 2: *(5 points)*

STEP 1: Picture

$$x^2 + y^2 + z^2 = 1 \Rightarrow \rho = 1$$
$$x^2 + y^2 + z^2 = 4 \Rightarrow \rho = 2$$
$$z = \sqrt{x^2 + y^2} \Rightarrow \phi = \frac{\pi}{4}$$



STEP 2: Inequalities

$$1 \leq \rho \leq 2$$
$$0 \leq \theta \leq \frac{\pi}{2}$$
$$0 \leq \phi \leq \frac{\pi}{4}$$

STEP 3: Integrate

$$
\begin{aligned}
&\int\int\int_E x^2+y^2+z^2\,dxdydz \\
=&\int_0^{\frac{\pi}{4}}\int_0^{\frac{\pi}{2}}\int_1^2 \rho^2\rho^2\sin(\phi)d\rho d\theta d\phi \\
=&\left(\int_1^2 \rho^4 d\rho\right)\left(\frac{\pi}{2}\right)\int_0^{\frac{\pi}{4}}\sin(\phi)d\phi \\
=&\left[\frac{\rho^5}{5}\right]_1^5\left(\frac{\pi}{2}\right)\left[-\cos(\phi)\right]_0^{\frac{\pi}{4}} \\
=&\left(\frac{2^5-1^5}{5}\right)\left(\frac{\pi}{2}\right)\left(-\frac{\sqrt{2}}{2}+1\right) \\
=&\frac{31\pi}{10}\left(1-\frac{\sqrt{2}}{2}\right)
\end{aligned}
$$