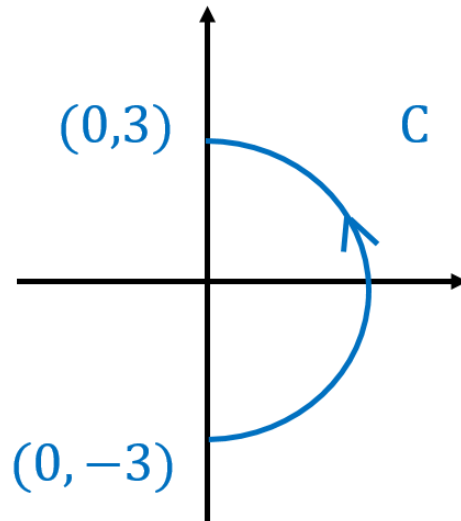


MATH 251 – QUIZ 9 – SOLUTIONS

Question 1: (5 points)

STEP 1: Picture



STEP 2: Parametrize

$$\begin{cases} x(t) = 3 \cos(t) \\ y(t) = 3 \sin(t) \\ -\frac{\pi}{2} \leq t \leq \frac{\pi}{2} \end{cases}$$

Date: Friday, November 19, 2021.

STEP 3: Integrate

$$\begin{aligned} & \int_C \mathbf{F} \cdot dr \\ &= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \mathbf{F}(r(t)) \cdot r'(t) dt \\ &= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \langle x(t)y(t), -(y(t))^2 \rangle \cdot \langle x'(t), y'(t) \rangle \\ &= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \langle 3 \cos(t)3 \sin(t), -9 \sin^2(t) \rangle \cdot \langle -3 \sin(t), 3 \cos(t) \rangle dt \\ &= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} -27 \cos(t) \sin^2(t) - 27 \sin^2(t) \cos(t) dt \\ &= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} -54 \cos(t) \sin^2(t) dt \\ &= \left[-\frac{54}{3} \sin^3(t) \right]_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \\ &= -18(1) + 18(-1) \\ &= -36 \end{aligned}$$

Question 2: (5 points)**STEP 1: Check F is conservative**

$$\begin{aligned} P_y &= (ye^x + 2x)_y = e^x \\ Q_x &= (e^x + 3y^2)_x = e^x \end{aligned}$$

Hence $P_y = Q_x$ ✓

STEP 2: Find f :

$$F = \nabla f$$
$$\langle ye^x + 2x, e^x + 3y^2 \rangle = \langle f_x, f_y \rangle$$

$$f_x = ye^x + 2x \Rightarrow f = \int ye^x + 2x dx = ye^x + x^2 + \text{JUNK}$$

$$f_y = e^x + 3y^2 \Rightarrow f = \int e^x + 3y^2 dy = ye^x + y^3 + \text{JUNK}$$

$$f(x, y) = ye^x + x^2 + y^3$$

STEP 3: Integrate:

$$\begin{aligned} \int_C F \cdot dr &= \int_C \nabla f \cdot dr \\ &= f(\text{End}) - f(\text{Start}) \\ &= f(2, 4) - f(1, 1) \\ &= 4e^2 + 2^2 + 4^3 - 1e^1 - 1^2 - 1^3 \\ &= 4e^2 - e + 66 \end{aligned}$$