APMA 1210 – FINAL EXAM

Name	
Banner ID	
Signature	

Instructions: Welcome to your Final Exam! You have 180 minutes to take this exam, for a total of 50 points. **You get 1 point just for filling out the info on this page**. No books, notes, calculators, or cellphones are allowed. **Please write in complete sentences if you can.** Remember that you are not only graded on your final answer, but also on your work. If you need to continue your work on scratch paper, please check the box "Work on scratch paper"

Academic Honesty Statement: With the signature above, I certify that the exam was taken by the person named and without any form of assistance and acknowledge that any form of cheating results in an automatic F in the course, and will be further subject to disciplinary consequences, pursuant to the Brown University Academic Code.

Date: Wednesday, December 14, 2022.

1. *(7 points)* Use the simplex method to solve

$$\begin{aligned} \max z = & 4x_1 + 3x_2 \\ \text{subject to } & 2x_1 + 3x_2 \leq 6 \\ & -3x_1 + 2x_2 \leq 3 \\ & 2x_1 + x_2 \leq 4 \\ & x_1 \geq 0 \\ & x_2 \geq 0 \end{aligned}$$

(Space to do Problem 1)

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2. *(7 points)* Welcome to Pokémon Stadium, this time Alakazam (P1) fighting against Bulbasaur (P2). Assume it's a zero-sum game and the gains matrix for Alakazam is:

Alakazam \ Bulbasaur	Razor Leaf	Poison Powder
Psybeam	-10	-8
Confusion	0	12
Psycho Cut	5	-10

Set up but do **NOT** solve the LP problems for both Alakazam and Bulbasaur, where Alakazam's strategy is $x = (x_1, x_2, x_3)$ and Bulbasaur's is $y = (y_1, y_2)$. No justification required, but write your final answer without matrices.

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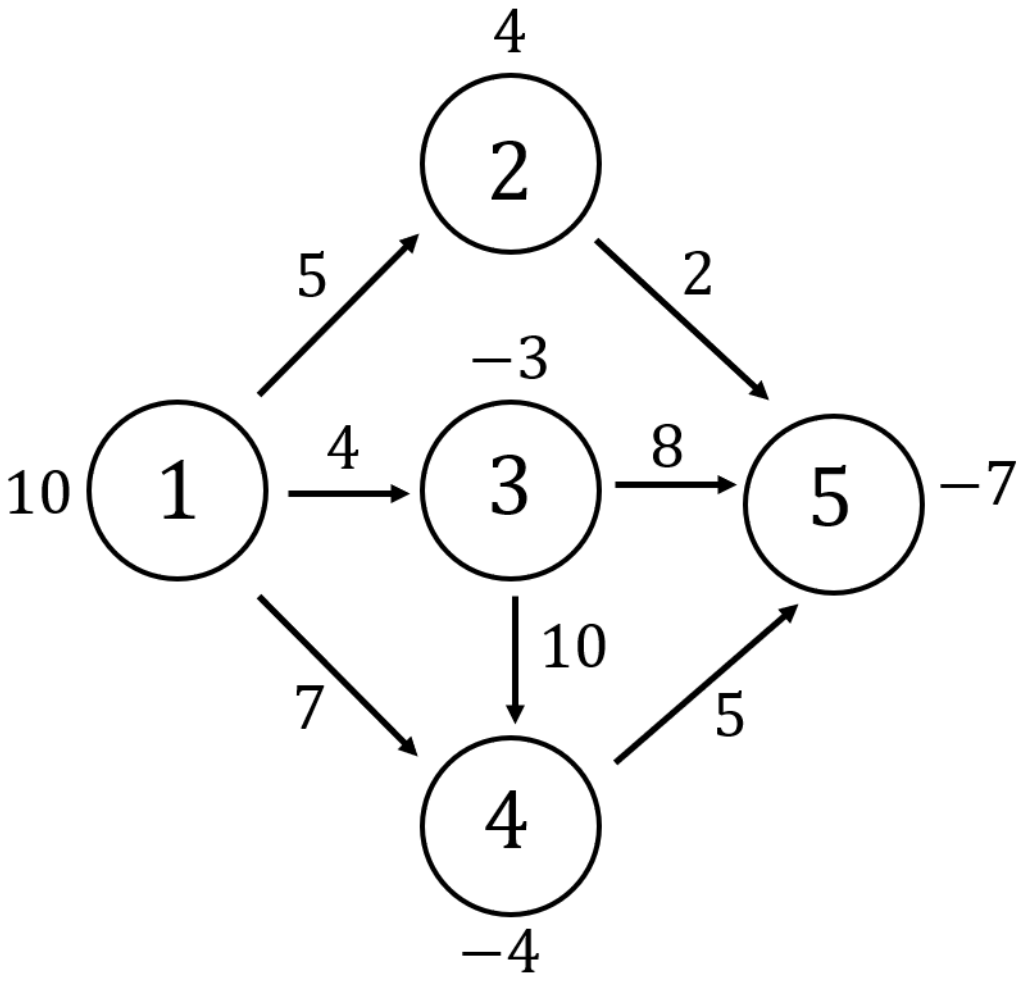
3. *(7 points)* Use the network simplex algorithm to find the optimal tree and optimal (minimal) total cost z of the following network. The numbers on the edges are the **COSTS**. Justify your answers



Note: Start with the tree $1 \to 2, 2 \to 5, 1 \to 3, 3 \to 4$

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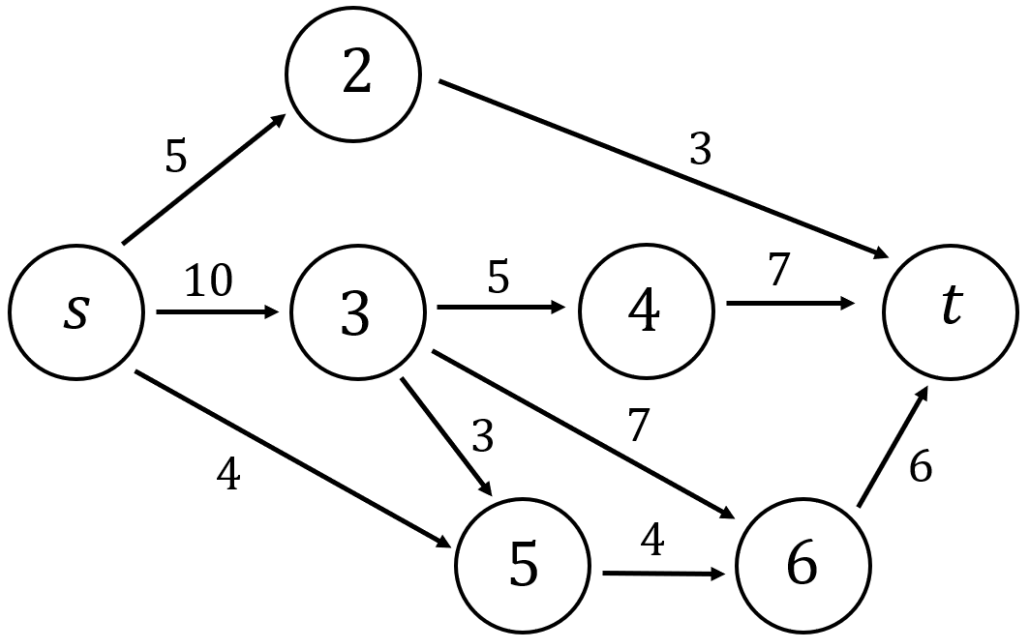
(Space to do Problem 3)

4. *(7 points)* Solve the max-flow problem (optimal edges and max flow) for the following network using residual graphs.

   Note: No need to find the min cut.



(Space to do Problem 4)

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5. *(7 points)* You are buying stocks from 4 different companies: Amazon (AMZN), BioNTech (BNTX), Costco (COST), and Disney (DIS). The net profit per share is \$16, \$22, \$12, and \$8 respectively. In addition, each company requires a one-time initiation fee of \$50, \$70, \$40, \$30. This fee is only applies if you invest in that company, and is independent of the number of stocks bought. The total budget you have to pay the initiation fee is \$140, and you can only buy at most 100 shares overall. Moreover:

   (1) You must invest from exactly three companies.

   (2) If you invest in BNTX, then you must also invest in AMZN

   (3) If you invest in BNTX, then you cannot invest in DIS

   Set up, but do **NOT** solve, an IP problem (decision variables, objective function, constraints) whose solution will tell you how to maximize your revenue (profit minus initiation fee) obtained from the investments.

   Hint: For (3), how many companies at most among BNTX and DIS can you invest in?

6. *(7 points)* Solve the following minimization problem (min point and value) where x_1 and x_2 are real numbers

$$\min z = (x_1)^2 + 3(x_2)^2 - (x_1)(x_2) - 3(x_1) - 4(x_2) + 8$$

Note: To justify your answer, please use both eigenvalues **AND** leading principal minors.

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7. *(7 points)* Show that if $f : \mathbb{R} \to \mathbb{R}$ is convex and $f(0) \leq 0$ then for all $a, b > 0$ we have

$$f(a) + f(b) \leq f(a + b)$$

Hint: Notice $a = \left(\frac{a}{a+b}\right)(a+b) + \left(\frac{b}{a+b}\right)0$

Apply f to this equation and explain why you can use convexity. Find a similar identity for $f(b)$ and conclude.

(Scratch paper)