APMA 0350 – FINAL EXAM – SOLUTIONS

1.

$$2t\,(y') - 3y = 9t^3$$

STEP 1: Standard form

$$y' - \left(\frac{3}{2t}\right)y = \left(\frac{9t^3}{2t}\right) = \left(\frac{9}{2}\right)t^2$$

STEP 2: Integrating factors

$$e^{\int -\frac{3}{2t}dt} = e^{-\frac{3}{2}\int \frac{1}{t}dt} = e^{-\frac{3}{2}\ln(t)} = t^{-\frac{3}{2}}$$

STEP 3: Multiply by $t^{-\frac{3}{2}}$

$$\begin{aligned}
t^{-\frac{3}{2}}\left(y' - \left(\frac{3}{2t}\right)y\right) &= \left(t^{-\frac{3}{2}}\right)\left(\frac{9}{2}\right)t^2 \\
\left(t^{-\frac{3}{2}}y\right)' &= \left(\frac{9}{2}\right)t^{\frac{1}{2}} \\
t^{-\frac{3}{2}}y &= \int \left(\frac{9}{2}\right)t^{\frac{1}{2}}dt \\
t^{-\frac{3}{2}}y &= \frac{9}{2}\left(\left(\frac{2}{3}\right)t^{\frac{3}{2}} + C\right) \\
t^{-\frac{3}{2}}y &= 3\,t^{\frac{3}{2}} + \underbrace{\frac{9}{2}C}_{C} \\
y &= 3t^{\frac{3}{2}}\,t^{\frac{3}{2}} + Ct^{\frac{3}{2}} \\
y &= 3t^3 + Ct^{\frac{3}{2}}
\end{aligned}$$

STEP 4: Answer

$$y = 3t^3 + Ct^{\frac{3}{2}}$$

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2.

$$\begin{cases} y' = & e^{2t+4y} \\ y(0) = & 0 \end{cases}$$

STEP 1: Separation of variables

$$\begin{aligned}
\frac{dy}{dt} =& e^{2t}e^{4y} \\
dy =& e^{2t}e^{4y}dt \\
e^{-4y}dy =& e^{2t}dt \\
\int e^{-4y}dy =& \int e^{2t}dt \\
-\frac{1}{4}e^{-4y} =& \frac{1}{2}e^{2t} + C \\
e^{-4y} =& -4\left(\frac{1}{2}e^{2t}\right) + \underbrace{(-4C)}_{C} \\
e^{-4y} =& -2e^{2t} + C \\
-4y =& \ln\left(-2e^{2t} + C\right) \\
y =& -\frac{1}{4}\ln\left(-2e^{2t} + C\right)
\end{aligned}$$

STEP 2: Initial Condition

$$\begin{aligned}
y(0) =& 0 \\
-\frac{1}{4}\ln\left(-2e^{0} + C\right) =& 0 \\
\ln\left(-2 + C\right) =& 0 \\
-2 + C =& e^{0} = 1 \\
C =& 1 + 2 = 3
\end{aligned}$$

STEP 3: Answer

$$y = -\frac{1}{4}\ln\left(-2e^{2t} + 3\right)$$

3.

$$\mathbf{x}' = A\mathbf{x} \qquad A = \begin{bmatrix} 1 & -3 \\ -2 & 2 \end{bmatrix} \qquad \mathbf{x}(0) = \begin{bmatrix} 5 \\ 0 \end{bmatrix}$$

STEP 1: Eigenvalues

$$\begin{aligned} \left|A - \lambda I\right| = &\begin{vmatrix} 1-\lambda & -3 \\ -2 & 2-\lambda \end{vmatrix} \\ =&(1-\lambda)(2-\lambda) - (-3)(-2) \\ =&2 - \lambda - 2\lambda + \lambda^2 - 6 \\ =&\lambda^2 - 3\lambda - 4 \\ =&(\lambda - 4)(\lambda + 1) = 0 \end{aligned}$$

Which gives $\lambda = 4$ or $\lambda = -1$

STEP 2: $\lambda = 4$

$$\text{Nul } (A - 4I) = \left[\begin{array}{cc|c} 1-4 & -3 & 0 \\ -2 & 2-4 & 0 \end{array}\right] = \left[\begin{array}{cc|c} -3 & -3 & 0 \\ -2 & -2 & 0 \end{array}\right] \longrightarrow \left[\begin{array}{cc|c} 1 & 1 & 0 \\ 0 & 0 & 0 \end{array}\right]$$

Hence $x + y = 0 \Rightarrow y = -x$

$$\mathbf{x} = \begin{bmatrix} x \\ -x \end{bmatrix} = x \begin{bmatrix} 1 \\ -1 \end{bmatrix} \rightsquigarrow \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

STEP 3: $\lambda = -1$

$$\text{Nul } (A - (-1)I) = \left[\begin{array}{cc|c} 1-(-1) & -3 & 0 \\ -2 & 2-(-1) & 0 \end{array}\right] = \left[\begin{array}{cc|c} 2 & -3 & 0 \\ -2 & 3 & 0 \end{array}\right] \longrightarrow \left[\begin{array}{cc|c} 2 & -3 & 0 \\ 0 & 0 & 0 \end{array}\right]$$

Hence $2x - 3y = 0$. For example $x = 3$ and $y = 2$ works and so

$$\mathbf{x} = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$$

STEP 4: Solution

$\lambda = 4 \rightsquigarrow \begin{bmatrix} 1 \\ -1 \end{bmatrix}$ and $\lambda = -1 \rightsquigarrow \begin{bmatrix} 3 \\ 2 \end{bmatrix}$ and so the gen solution is

$$\mathbf{x}(t) = C_1 e^{4t} \begin{bmatrix} 1 \\ -1 \end{bmatrix} + C_2 e^{-t} \begin{bmatrix} 3 \\ 2 \end{bmatrix}$$

STEP 5: Initial Condition

$$\begin{aligned}
\mathbf{x}(0) &= \begin{bmatrix} 5 \\ 0 \end{bmatrix} \\
C_1 e^0 \begin{bmatrix} 1 \\ -1 \end{bmatrix} + C_2 e^0 \begin{bmatrix} 3 \\ 2 \end{bmatrix} &= \begin{bmatrix} 5 \\ 0 \end{bmatrix} \\
C_1 \begin{bmatrix} 1 \\ -1 \end{bmatrix} + C_2 \begin{bmatrix} 3 \\ 2 \end{bmatrix} &= \begin{bmatrix} 5 \\ 0 \end{bmatrix} \\
\begin{bmatrix} 1 & 3 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} C_1 \\ C_2 \end{bmatrix} &= \begin{bmatrix} 5 \\ 0 \end{bmatrix}
\end{aligned}$$

$$\left[\begin{array}{cc|c} 1 & 3 & 5 \\ -1 & 2 & 0 \end{array}\right] \to \left[\begin{array}{cc|c} 1 & 3 & 5 \\ 0 & 5 & 5 \end{array}\right] \to \left[\begin{array}{cc|c} 1 & 3 & 5 \\ 0 & 1 & 1 \end{array}\right] \to \left[\begin{array}{cc|c} 1 & 0 & 2 \\ 0 & 1 & 1 \end{array}\right]$$

$$\begin{cases} C_1 = 2 \\ C_2 = 1 \end{cases}$$

STEP 5: Solution

$$\mathbf{x}(t) = 2e^{4t} \begin{bmatrix} 1 \\ -1 \end{bmatrix} + 1e^{-t} \begin{bmatrix} 3 \\ 2 \end{bmatrix} = e^{4t} \begin{bmatrix} 2 \\ -2 \end{bmatrix} + e^{-t} \begin{bmatrix} 3 \\ 2 \end{bmatrix}$$

4.
$$\begin{cases} x' = & 6x - 2x^2 - xy \\ y' = & 4y - y^2 - xy \end{cases}$$

STEP 1: Equilibrium Solutions

$$\begin{cases} x' = & 6x - 2x^2 - xy = 0 \\ y' = & 4y - y^2 - xy = 0 \end{cases} \Rightarrow \begin{cases} x(6 - 2x - y) = & 0 \\ y(4 - y - x) = & 0 \end{cases}$$

Case 1: $x = 0$

Then either $y = 0 \rightsquigarrow (0, 0)$

Or $4 - y - x = 0 \Rightarrow 4 - y = 0 \Rightarrow y = 4 \rightsquigarrow (0, 4)$

Case 2: $6 - 2x - y = 0$

Either $y = 0$, so $6 - 2x = 0 \Rightarrow x = 3 \rightsquigarrow (3, 0)$

Or $4 - y - x = 0$ and so we have to solve

$$\begin{cases} 6 - 2x - y = & 0 \\ 4 - y - x = & 0 \end{cases}$$

$4 - y - x = 0 \Rightarrow y = 4 - x$

$$6 - 2x - (4 - x) = 0 \Rightarrow 2 - x = 0 \Rightarrow x = 2$$

And $y = 4 - x = 2 - 2 = 2 \rightsquigarrow (2, 2)$

Equilibrium Points: $(0, 0), (0, 4), (3, 0), (2, 2)$

STEP 2: Classification

$$\nabla F(x, y) = \begin{bmatrix} 6 - 4x - y & -x \\ -y & 4 - 2y - x \end{bmatrix}$$

Case 1: $(0, 0)$

$$\nabla F(0,0) = \begin{bmatrix} 6-0-0 & 0 \\ 0 & 4-0-0 \end{bmatrix} = \begin{bmatrix} 6 & 0 \\ 0 & 4 \end{bmatrix}$$

$\lambda = 6 > 0$ and $\lambda = 4 > 0$, so $(0,0)$ is **unstable**

Case 2: $(0,4)$

$$\nabla F(0,4) = \begin{bmatrix} 6-0-4 & 0 \\ -4 & 4-2(4)-0 \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ -4 & -4 \end{bmatrix}$$

$\lambda = 2 > 0$ and $\lambda = -4 < 0$, so $(0,4)$ is a **saddle**

Case 3: $(3,0)$

$$\nabla F(3,0) = \begin{bmatrix} 6-4(3)-0 & -3 \\ -0 & 4-0-3 \end{bmatrix} = \begin{bmatrix} -6 & -3 \\ 0 & 1 \end{bmatrix}$$

$\lambda = -6 < 0$ and $\lambda = 1 > 0$ so $(3,0)$ is a **saddle**

Case 4: $(2,2)$

$$\nabla F(2,2) = \begin{bmatrix} 6-4(2)-2 & -2 \\ -2 & 4-2(2)-2 \end{bmatrix} = \begin{bmatrix} -4 & -2 \\ -2 & -2 \end{bmatrix}$$

Eigenvalues:

$$\begin{aligned}
\det(A-\lambda I) &= \begin{vmatrix} -4-\lambda & -2 \\ -2 & -2-\lambda \end{vmatrix} \\
&= (-4-\lambda)(-2-\lambda) - (-2)(-2) \\
&= 8 + 4\lambda + 2\lambda + \lambda^2 - 4 \\
&= \lambda^2 + 6\lambda + 4 \\
&= (\lambda+3)^2 - 9 + 4 \\
&= (\lambda+3)^2 - 5
\end{aligned}$$

$$(\lambda+3)^2 = 5 \Rightarrow \lambda = -3 \pm \sqrt{5}$$

$\lambda = -3 - \sqrt{5} < 0$ and $\lambda = -3 + \sqrt{5} < 0$ since $3 = \sqrt{9} > \sqrt{5}$

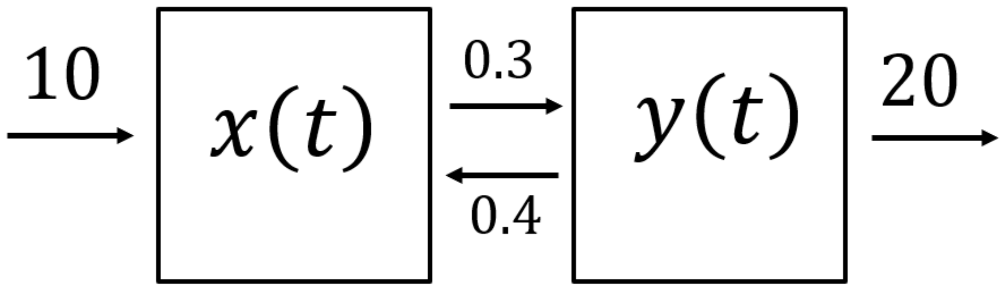
Hence $(2,2)$ is **stable**

STEP 3: Answer:

$$\begin{aligned}&(0,0) \text{ unstable}\\&(3,0) \text{ saddle}\\&(0,4) \text{ saddle}\\&(2,2) \text{ stable}\end{aligned}$$

5.

Notice that $100 - 60 = 40\%$ of cars go from Baltimore to Albuquerque



$$\begin{cases} x'(t) = & -0.3x(t) + 0.4y(t) + 10 \\ y'(t) = & 0.3x(t) - 0.4y(t) - 20 \end{cases}$$

Which you can write as $\mathbf{x}'(t) = A\mathbf{x}(t) + \mathbf{f}(t)$ where

$$A = \begin{bmatrix} -0.3 & 0.4 \\ 0.3 & -0.4 \end{bmatrix} \qquad \mathbf{f} = \begin{bmatrix} 10 \\ -20 \end{bmatrix}$$

6.

$$\begin{cases} y'' = \lambda y \\ y(0) = 0 \\ y'(3) = 0 \end{cases}$$

Auxiliary Equation: $r^2 = \lambda$

Case 1: $\lambda > 0$

Then $r^2 = \lambda = \omega^2$ and so $r = \pm\omega$

$$y = Ae^{\omega t} + Be^{-\omega t}$$

$$y(0) = A + B = 0 \Rightarrow B = -A$$

$$y = Ae^{\omega t} - Ae^{-\omega t}$$

$$y' = A\omega e^{\omega t} - A(-\omega)e^{-\omega t}$$

$$\begin{aligned} y'(3) &= 0 \\ A\omega e^{3\omega} + A\omega e^{-3\omega} &= 0 \\ \cancel{A\omega} e^{3\omega} &= -\cancel{A\omega} e^{-3\omega} \\ \underbrace{e^{3\omega}}_{>0} &= \underbrace{-e^{-3\omega}}_{<0} \end{aligned}$$

Which is a contradiction $\Rightarrow\Leftarrow$

Case 2: $\lambda = 0$

Aux: $r^2 = 0 \Rightarrow r = 0$ (repeated twice)

$$y = A + Bt$$

$$y(0) = 0 \Rightarrow A = 0$$

$$y = Bt$$

$$y'(t) = B$$

$$y'(3) = 0 \Rightarrow B = 0$$

But then in this case $y = 0 \Rightarrow\Leftarrow$

Case 3: $\lambda < 0$

In this case $\lambda = -\omega^2$ where $\omega > 0$

Aux: $r^2 = \lambda = -\omega^2 \Rightarrow r = \pm \omega i$

$$y = A\cos(\omega t) + B\sin(\omega t)$$

$$y(0) = A = 0$$
$$y = B\sin(\omega t)$$
$$y' = \omega B\cos(\omega t)$$

$$\begin{aligned} y'(3) &= 0 \\ \not{\omega}\not{B}\cos(3\omega) &= 0 \\ \cos(3\omega) &= 0 \\ 3\omega &= \frac{\pi}{2} + \pi m \\ \omega &= \left(\frac{\pi}{6}\right) + \left(\frac{\pi}{3}\right) m \qquad m = 0, 1, 2, \cdots \end{aligned}$$

Answer:

Eigenvalues:

$$\lambda = -\omega^2 = -\left(\frac{\pi}{6} + \left(\frac{\pi}{3}\right) m\right)^2 \qquad m = 0, 1, 2, \cdots$$

Eigenfunctions:

$$y = \sin(\omega t) = \sin\left(\left(\frac{\pi}{6} + \left(\frac{\pi}{3}\right) m\right) t\right) \qquad m = 0, 1, 2 \cdots$$

7.

$$2y'' + 2y = 4\sec(t)$$

STEP 1: Standard form: $y'' + y = 2\sec(t)$

STEP 2: Homogeneous Solution

$$r^2 + 1 = 0 \Rightarrow r = \pm i \Rightarrow y_0 = A\cos(t) + B\sin(t)$$

STEP 3: Variation of Parameters

$$y_p = u(t)\cos(t) + v(t)\sin(t)$$

$$\begin{bmatrix} \cos(t) & \sin(t) \\ -\sin(t) & \cos(t) \end{bmatrix} \begin{bmatrix} u'(t) \\ v'(t) \end{bmatrix} = \begin{bmatrix} 0 \\ 2\sec(t) \end{bmatrix}$$

STEP 4: Cramer's Rule

$$u'(t) = \frac{\begin{vmatrix} 0 & \sin(t) \\ 2\sec(t) & \cos(t) \end{vmatrix}}{\begin{vmatrix} \cos(t) & \sin(t) \\ -\sin(t) & \cos(t) \end{vmatrix}} = \frac{0 - \sin(t)2\sec(t)}{\cos^2(t) + \sin^2(t)} = -\sin(t)\left(\frac{2}{\cos(t)}\right) = -2\tan(t)$$

$$v'(t) = \frac{\begin{vmatrix} \cos(t) & 0 \\ -\sin(t) & 2\sec(t) \end{vmatrix}}{1} = 2\sec(t)\cos(t) = \left(\frac{2}{\cos(t)}\right)\cos(t) = 2$$

STEP 5: Integrate

$$u'(t) = -2\tan(t) \Rightarrow u(t) = \int -2\tan(t)dt = -2\ln|\sec(t)|$$

$$v'(t) = 2 \Rightarrow v(t) = \int 2dt = 2t$$

STEP 6:

$$y_p = u(t)\cos(t) + v(t)\sin(t) = (-2\ln|\sec(t)|)\cos(t) + 2t\sin(t)$$

$$y = y_0 + y_p = A\cos(t) + B\sin(t) - (2\ln|\sec(t)|)\cos(t) + 2t\sin(t)$$

Note: It's also acceptable to use $2(\ln|\cos(t)|)\cos(t)$

8.

$$\mathbf{x}' = A\mathbf{x} + \mathbf{f} \quad A = \begin{bmatrix} 2 & -5 \\ 1 & 4 \end{bmatrix} \qquad \mathbf{f} = \begin{bmatrix} 5e^{7t} \\ 7e^{7t} \end{bmatrix}$$

STEP 1: Eigenvalues

$$\begin{aligned} \det(A - \lambda I) = & \begin{vmatrix} 2-\lambda & -5 \\ 1 & 4-\lambda \end{vmatrix} \\ = & (2-\lambda)(4-\lambda) - (-5)(1) \\ = & 8 - 2\lambda - 4\lambda + \lambda^2 + 5 \\ = & \lambda^2 - 6\lambda + 13 \\ = & (\lambda - 3)^2 - 9 + 13 \\ = & (\lambda-3)^2 + 4 \end{aligned}$$

$$\lambda = 3 \pm 2i$$

$$\mathbf{f} = e^{7t} \begin{bmatrix} 5 \\ 7 \end{bmatrix} \rightsquigarrow \lambda = 7$$

This doesn't coincide, so there is no resonance.

STEP 2: Undetermined coefficients

$$\mathbf{x_p}(t) = e^{7t} \begin{bmatrix} A \\ B \end{bmatrix} = \begin{bmatrix} Ae^{7t} \\ Be^{7t} \end{bmatrix}$$

$$\begin{aligned} \mathbf{x_p}' = & A\mathbf{x_p} + \mathbf{f} \\ \begin{bmatrix} 7Ae^{7t} \\ 7Be^{7t} \end{bmatrix} = & \begin{bmatrix} 2 & -5 \\ 1 & 4 \end{bmatrix} \begin{bmatrix} Ae^{7t} \\ Be^{7t} \end{bmatrix} + \begin{bmatrix} 5e^{7t} \\ 7e^{7t} \end{bmatrix} \\ \begin{bmatrix} 7A\cancel{e^{7t}} \\ 7B\cancel{e^{7t}} \end{bmatrix} = & \begin{bmatrix} 2A\cancel{e^{7t}} - 5B\cancel{e^{7t}} \\ A\cancel{e^{7t}} + 4B\cancel{e^{7t}} \end{bmatrix} + \begin{bmatrix} 5\cancel{e^{7t}} \\ 7\cancel{e^{7t}} \end{bmatrix} \\ \begin{bmatrix} 7A \\ 7B \end{bmatrix} = & \begin{bmatrix} 2A - 5B + 5 \\ A + 4B + 7 \end{bmatrix} \end{aligned}$$

Therefore we need to solve the system

$$\begin{cases} 7A = 2A - 5B + 5 \\ 7B = A + 4B + 7 \end{cases} \Rightarrow \begin{cases} 5A = -5B + 5 \\ 3B = A + 7 \end{cases}$$

The first equation gives $A = -B + 1$ and plugging it into the second equation, we get

$$\begin{aligned} 3B =& (-B + 1) + 7 \\ 4B =& 8 \\ B =& 2 \end{aligned}$$

And hence $A = -2 + 1 = -1$

$$\textbf{Answer: } \mathbf{x_p}(t) = e^{7t} \begin{bmatrix} -1 \\ 2 \end{bmatrix}$$

9.

$$\begin{cases} y'' - 5y' + 6y = & 5\delta(t-3) \star \delta(t-4) \\ y(0) = & 0 \\ y'(0) = & 2 \end{cases}$$

STEP 1:

$$\begin{aligned} \mathcal{L}\left\{y''\right\} - 5\mathcal{L}\left\{y'\right\} + 6\mathcal{L}\left\{y\right\} =& 5\mathcal{L}\left\{\delta(t-3) \star \delta(t-4)\right\} \\ \left(s^2\mathcal{L}\left\{y\right\} - sy(0) - y'(0)\right) - 5\left(s\mathcal{L}\left\{y\right\} - y(0)\right) + 6\mathcal{L}\left\{y\right\} =& 5\mathcal{L}\left\{\delta(t-3)\right\}\mathcal{L}\left\{\delta(t-4)\right\} \\ s^2\mathcal{L}\left\{y\right\} - 2 - 5s\mathcal{L}\left\{y\right\} + 6\mathcal{L}\left\{y\right\} =& 5e^{-3s}e^{-4s} \\ \left(s^2 - 5s + 6\right)\mathcal{L}\left\{y\right\} =& 2 + 5e^{-7s} \end{aligned}$$

$$\mathcal{L}\left\{y\right\} = \left(\frac{1}{s^2 - 5s + 6}\right)\left(2 + 5e^{-7s}\right)$$

STEP 2: Partial Fractions

Note: $s^2 - 5s + 6 = (s-2)(s-3)$

$$\begin{aligned} \frac{1}{s^2 - 5s + 6} =& \frac{A}{s-2} + \frac{B}{s-3} \\ =& \frac{A(s-3) + B(s-2)}{(s-2)(s-3)} \\ =& \frac{As - 3A + Bs - 2B}{s^2 - 5s + 6} \\ =& \frac{(A+B)s + (-3A - 2B)}{s^2 - 5s + 6} \end{aligned}$$

$$\begin{cases} A + B = & 0 \\ -3A - 2B = & 1 \end{cases}$$

The first equation gives $B = -A$ and the second becomes

$$-3A - 2(-A) = 1 \Rightarrow -3A + 2A = 1 \Rightarrow -A = 1 \Rightarrow A = -1$$

$$B = -A = 1$$

$$\frac{1}{s^2 - 5s + 6} = -\frac{1}{s-2} + \frac{1}{s-3}$$

STEP 3:

$$\mathcal{L}\left\{y\right\} = \left(-\frac{1}{s-2} + \frac{1}{s-3}\right)\left(2 + 5e^{-7s}\right)$$
$$= \mathcal{L}\left\{-e^{2t} + e^{3t}\right\}\left(2 + 5e^{-7s}\right)$$
$$= \mathcal{L}\left\{-2e^{2t} + 2e^{3t} + \left(-5e^{2(t-7)} + 5e^{3(t-7)}\right)u_7(t)\right\}$$

STEP 4: Answer:

$$y = -2e^{2t} + 2e^{3t} + \left(-5e^{2(t-7)} + 5e^{3(t-7)}\right)u_7(t)$$

Note: Ok to write this as

$$y = 2h(t) + 5h(t-7)u_7(t) \text{ where } h(t) = -e^{2t} + e^{3t}$$

10.

$$\left(\frac{2}{s^3}\right) \times \left(\frac{s+2}{s^2+4s+13}\right) e^{-5s}$$

STEP 1: First function

$$\frac{2!}{s^3} = \mathcal{L}\left\{t^2\right\}$$

STEP 2: Second function

$$\text{Look at } \frac{s+2}{s^2+4s+13} = \frac{s+2}{(s+2)^2 - 4 + 13} = \frac{s+2}{(s+2)^2 + 9}$$

$$\text{Shifted version by } -2 \text{ units of } \frac{s}{s^2+9} = \mathcal{L}\left\{\cos(3t)\right\}$$

$$\text{Therefore } \frac{s+2}{s^2+4s+13} = \mathcal{L}\left\{e^{-2t}\cos(3t)\right\}$$

$$\left(\frac{s+2}{s^2+4s+13}\right) e^{-5s} = \mathcal{L}\left\{e^{-2t}\cos(3t)\right\} e^{-5s} = \mathcal{L}\left\{e^{-2(t-5)}\cos(3(t-5))u_5(t)\right\}$$

STEP 3: Answer: Just the convolution of the two

$$\int_0^t (t-\tau)^2 e^{-2(\tau-5)}\cos(3(\tau-5))u_5(\tau)d\tau$$

Note: It would also be ok to put the e^{-5s} term with the $\frac{2}{s^3}$