APMA 0350 – FINAL EXAM

Name	
Banner ID	
Signature	

Instructions: Welcome to your final. You have 180 minutes to take this exam, for a total of 50 points. No books, notes, cheat sheets, calculators, or cellphones are allowed. **Please put your answers in the boxes provided.** Remember that you are not only graded on your final answer, but also on your work. If you need to continue your work on scratch paper, please check the box "Work on scratch paper"

Academic Honesty Statement: With the signature above, I certify that the exam was taken by the person named and without any form of assistance and acknowledge that any form of cheating results in an automatic F in the course, and will be further subject to disciplinary consequences, pursuant to the Brown University Academic Code.

Date: Monday, December 19, 2022.

$f(t)$	$\mathcal{L}\{f(t)\}$
1	$\frac{1}{s}$
e^{at}	$\frac{1}{s-a}$
t^n	$\frac{n!}{s^{n+1}}$
$\sin(at)$	$\frac{a}{s^2+a^2}$
$\cos(at)$	$\frac{s}{s^2+a^2}$
$u_c(t)$	$\frac{e^{-cs}}{s}$
$u_c(t)f(t-c)$	$e^{-cs}\mathcal{L}\{f(t)\}$
$e^{at}f(t)$	$F(s-a)$
$\delta(t-c)$	e^{-cs}
y'	$s\mathcal{L}\{y\} - y(0)$
y''	$s^2\mathcal{L}\{y\} - sy(0) - y'(0)$

$$(f \star g)(t) = \int_0^t f(t-\tau)g(\tau)d\tau$$

1. *(5 points)* Find the general solution of the ODE (here $t > 0$)

$$2t\,(y') - 3y = 9t^3$$

$$y = \boxed{}$$

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2. *(5 points)* Solve the following ODE

$$\begin{cases} y' = e^{2t+4y} \\ y(0) = 0 \end{cases}$$

$y =$	

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3. *(5 points)* Solve the following system $\mathbf{x}' = A\mathbf{x}$ where

$$A = \begin{bmatrix} 1 & -3 \\ -2 & 2 \end{bmatrix} \qquad \mathbf{x}(0) = \begin{bmatrix} 5 \\ 0 \end{bmatrix}$$

Simplify your answer. No need to draw a phase portrait.

$\mathbf{x}(t) =$	

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4. *(5 points)* Find and classify the equilibrium point(s) of

$$\begin{cases} x' = 6x - 2x^2 - xy \\ y' = 4y - y^2 - xy \end{cases}$$

Answer	

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5. *(5 points)* Suppose you're modeling the flow of cars between Albuquerque and Baltimore. Every day, we simultaneously have

- 10 thousand cars enter Albuquerque (not from Baltimore)
- 30% of the cars from Albuquerque move to Baltimore
- 60% of the cars from Baltimore do not move to Albuquerque
- 20 thousand cars exit Baltimore (not to Albuquerque)

Let $x(t)$ and $y(t)$ be the number of thousands of cars in Albuquerque and Baltimore respectively, where t is in days.

Set up but do **NOT** solve an ODE model for $x(t)$ and $y(t)$

Note: Write your answer in matrix form, and include a diagram similar to the chemical tank model. No justification required

Answer	$A =$	$\mathbf{f} =$

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6. *(5 points)* Find all the eigenvalues and corresponding eigenfunctions of the following ODE

$$\begin{cases} y'' = \lambda y \\ y(0) = 0 \\ y'(3) = 0 \end{cases}$$

$\lambda =$	
$y =$	

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7. *(5 points)* You may have heard of Elf on the Shelf, now get ready for Var of Par ☺

Use Var of Par to find the general solution of

$$2y'' + 2y = 4\sec(t)$$

$y =$	

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8. *(5 points)* Use undetermined coefficients to find a particular solution $\mathbf{x_p}$ to $\mathbf{x}' = A\mathbf{x} + \mathbf{f}$ where

$$A = \begin{bmatrix} 2 & -5 \\ 1 & 4 \end{bmatrix} \qquad \mathbf{f} = \begin{bmatrix} 5e^{7t} \\ 7e^{7t} \end{bmatrix}$$

Note: Don't forget to check for resonance.

$\mathbf{x_p}(t) =$	

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9. *(5 points)* Solve the following ODE

$$\begin{cases} y'' - 5y' + 6y = 5\delta(t-3) \star \delta(t-4) \\ y(0) = 0 \\ y'(0) = 2 \end{cases}$$

$y =$	

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10. *(5 points)* Find a function whose Laplace transform is

$$\left(\frac{2}{s^3}\right) \times \left(\frac{s+2}{s^2+4s+13}\right) e^{-5s}$$

Note: Write your answer as an integral

$f(t) =$	

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(Scratch Paper)