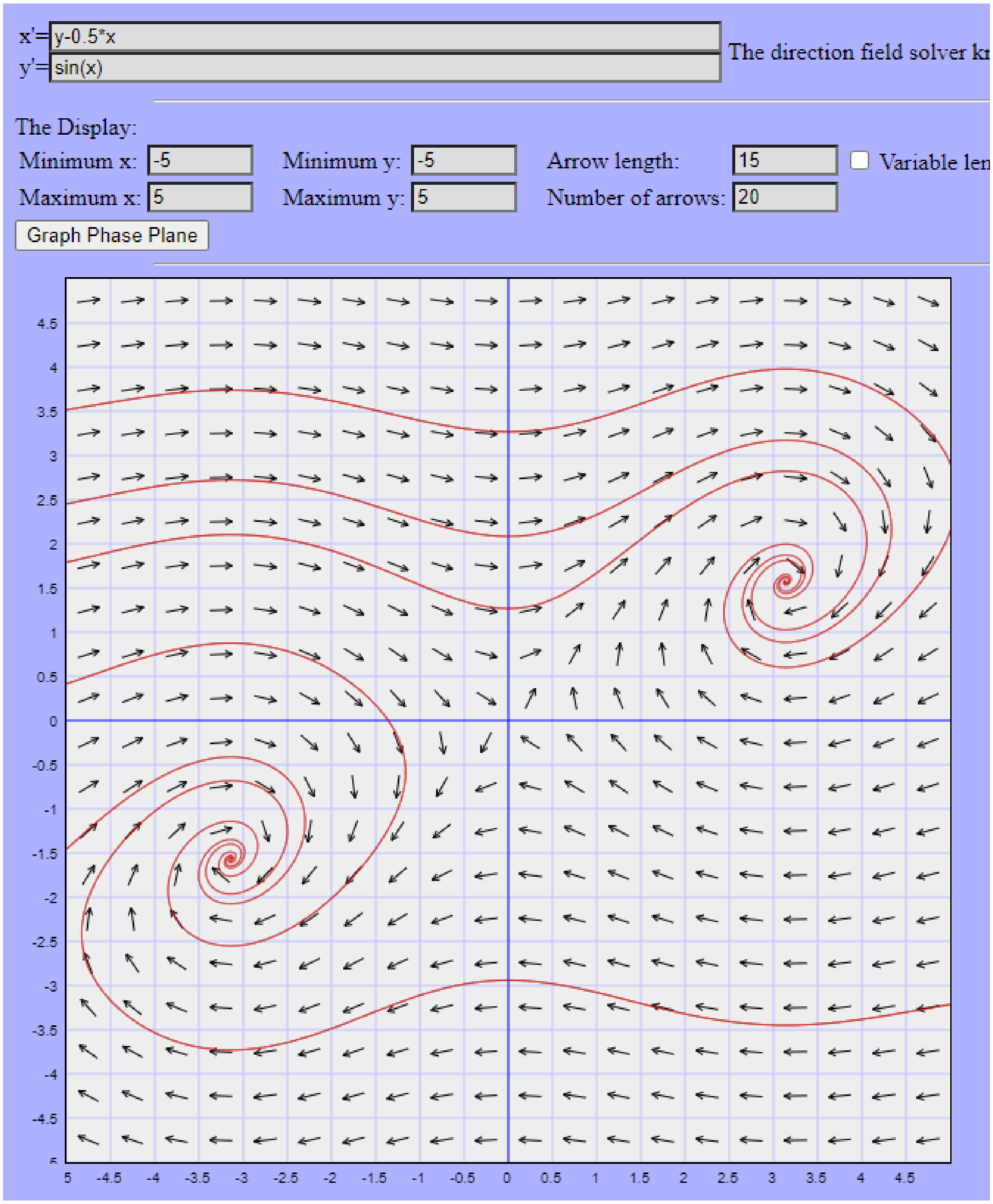
PEYAMSTAGRAM

Systems of ODE give you incredibly pretty graphs! This is why I fell in love with them ☺

Example 1:

$$\begin{cases} x' = -(0.5)x + y \\ y' = \sin(x) \end{cases}$$

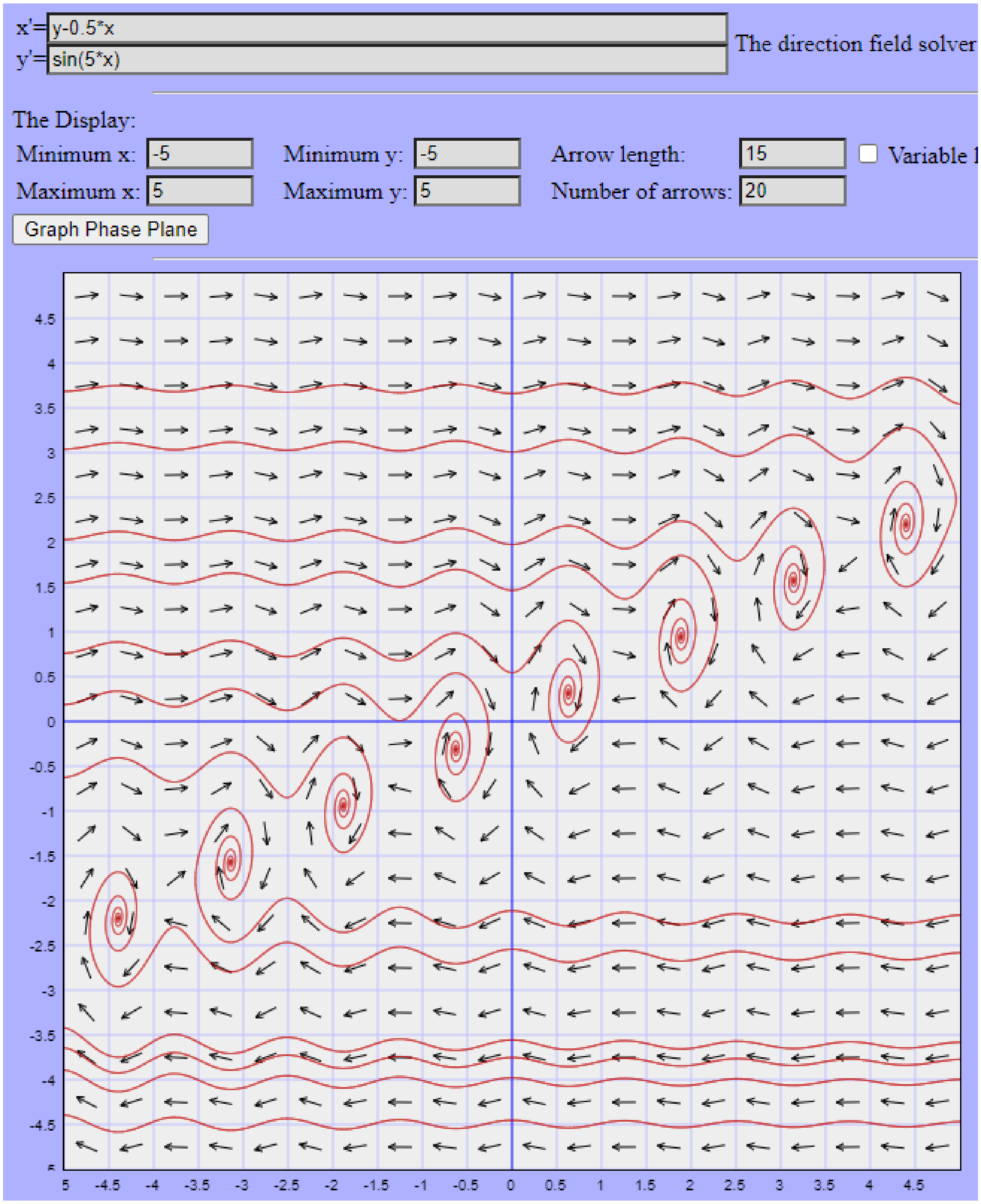

x'= y-0.5*x
y'= sin(x)
The direction field solver kr
The Display:
Minimum x: -5
Minimum y: -5
Arrow length: 15
Variable len
Maximum x: 5
Maximum y: 5
Number of arrows: 20
Graph Phase Plane

This is the ODE version of Van Gogh's *Starry Night*

If you want more sinks, no problem!

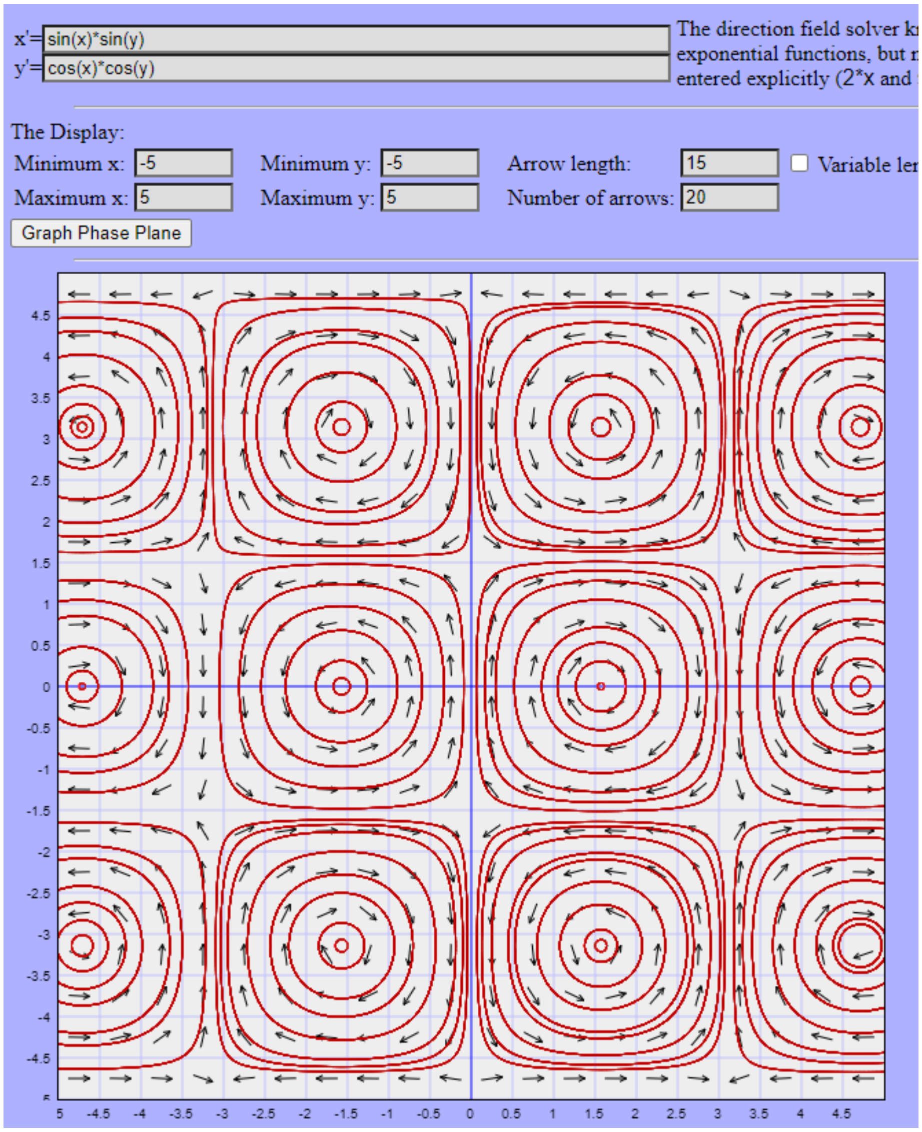
Example 2:

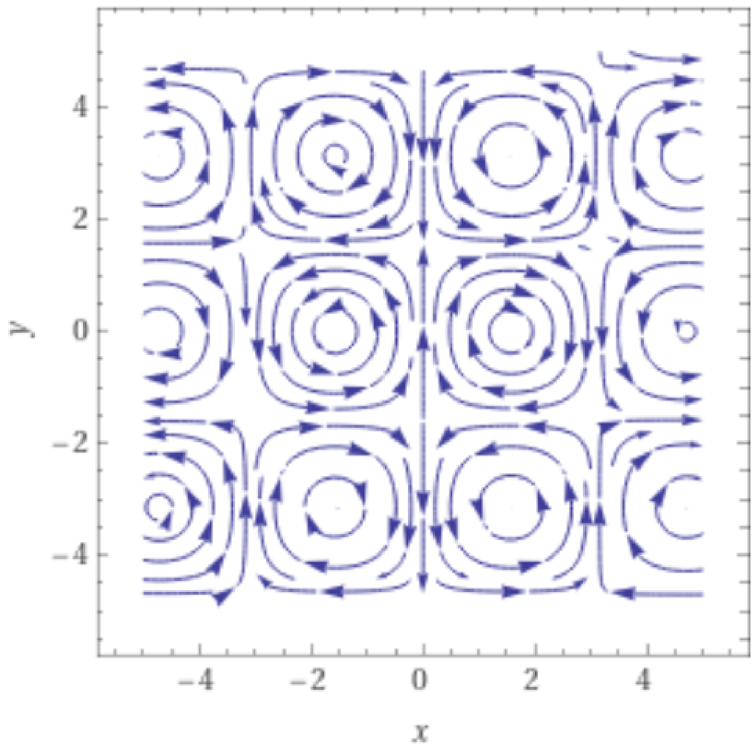
$$\begin{cases} x' = -(0.5)x + y \\ y' = \sin(5x) \end{cases}$$


x'= y-0.5*x
y'= sin(5*x)
The direction field solver
The Display:
Minimum x: -5
Maximum x: 5
Minimum y: -5
Maximum y: 5
Arrow length: 15
Number of arrows: 20
Variable l
Graph Phase Plane
4.5
4
3.5
3
2.5
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-2.5
-3
-3.5
-4
-4.5
5
-4.5
-4
-3.5
-3
-2.5
-2
-1.5
-1
-0.5
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5

Example 3: (Wishy-Washy ODE)

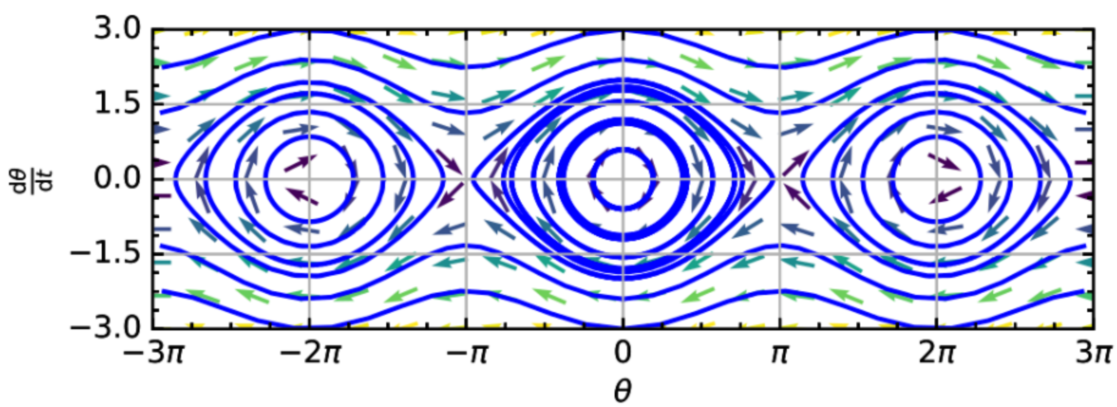
$$\begin{cases} x' = \sin(x)\sin(y) \\ y' = \cos(x)\cos(y) \end{cases}$$





Example 4:

Pendulum Equation

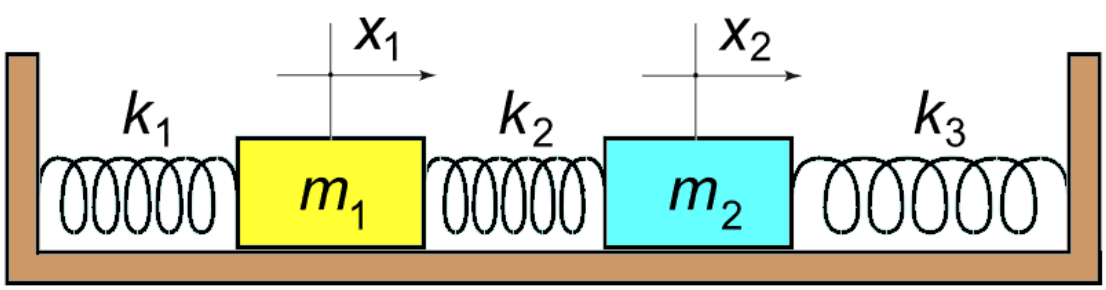


The next one is **by far** my favorite example, and is the reason why I got so hooked on ODEs

Example 5: (Coupled Harmonic Oscillator)

$$\begin{cases} x'' = -4x \\ y'' = -4y \end{cases}$$

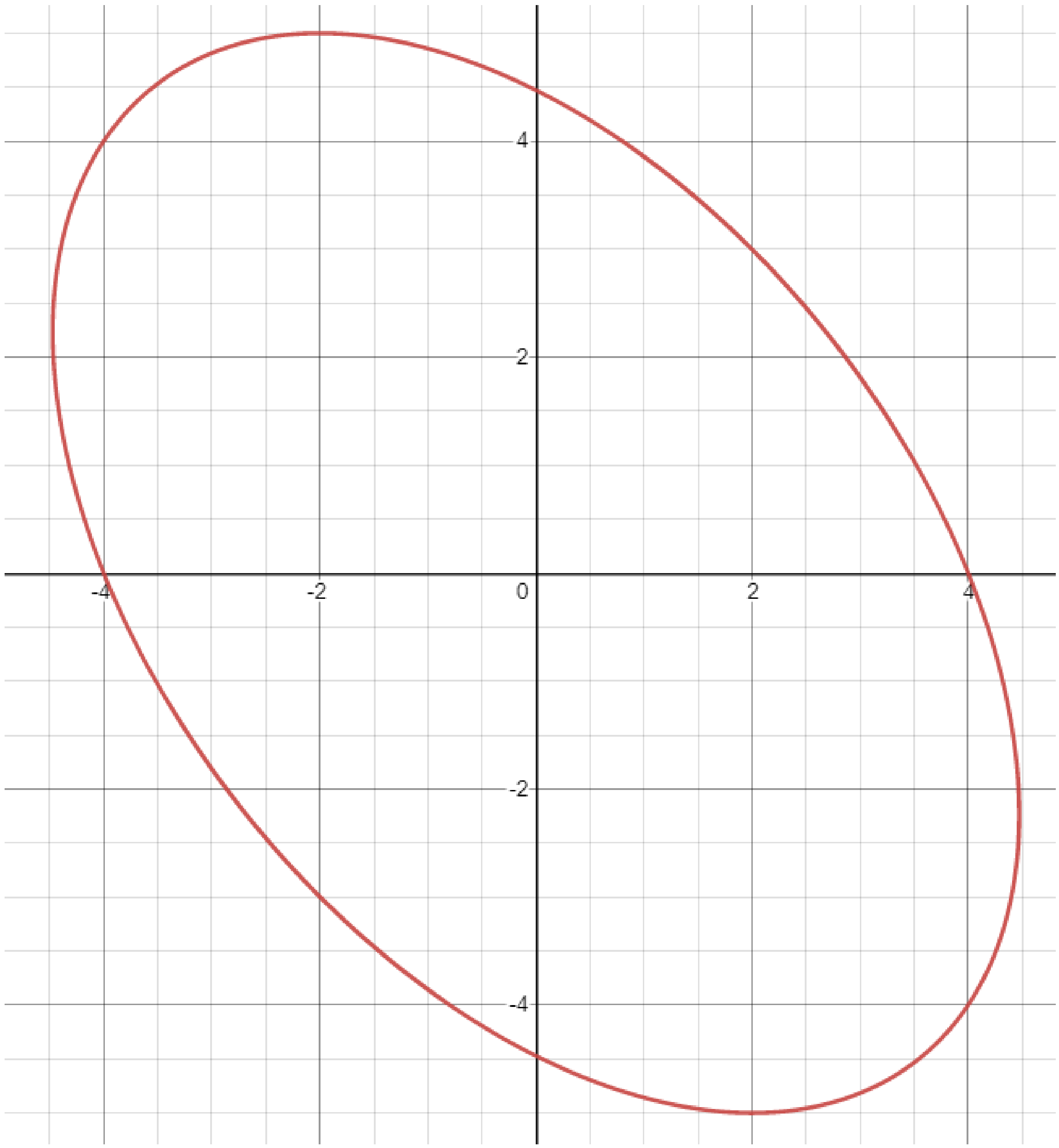
This is a simplified version of the double harmonic oscillator:



This system is uncoupled and can be solved separately:

$$\begin{cases} x(t) = A\cos(2t) + B\sin(2t) \\ y(t) = C\cos(2t) + D\sin(2t) \end{cases}$$

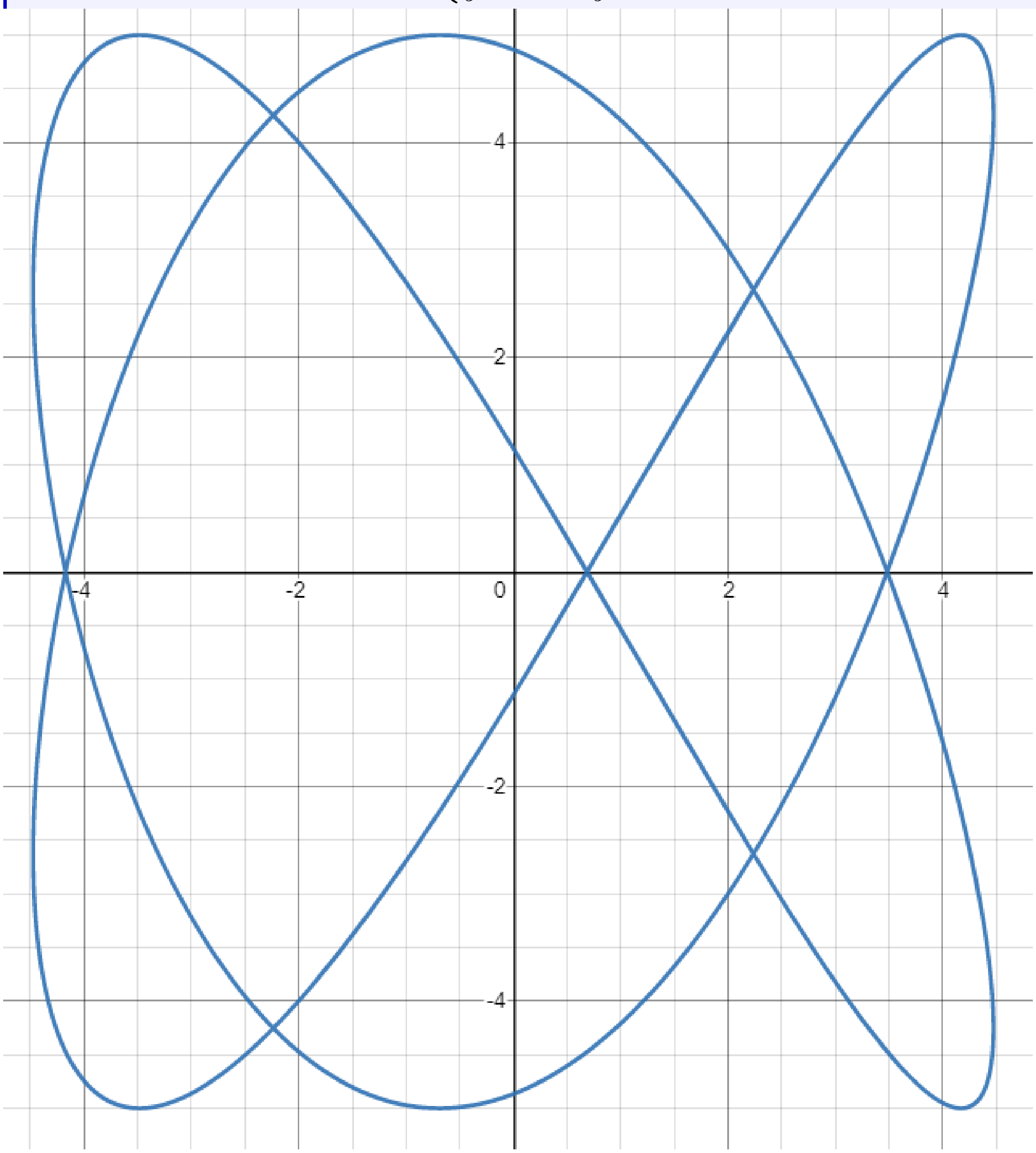
In this case the solution is nice, it lies on an ellipse:



What if we change the system a bit? It will look crazier, but it's still a closed curve!

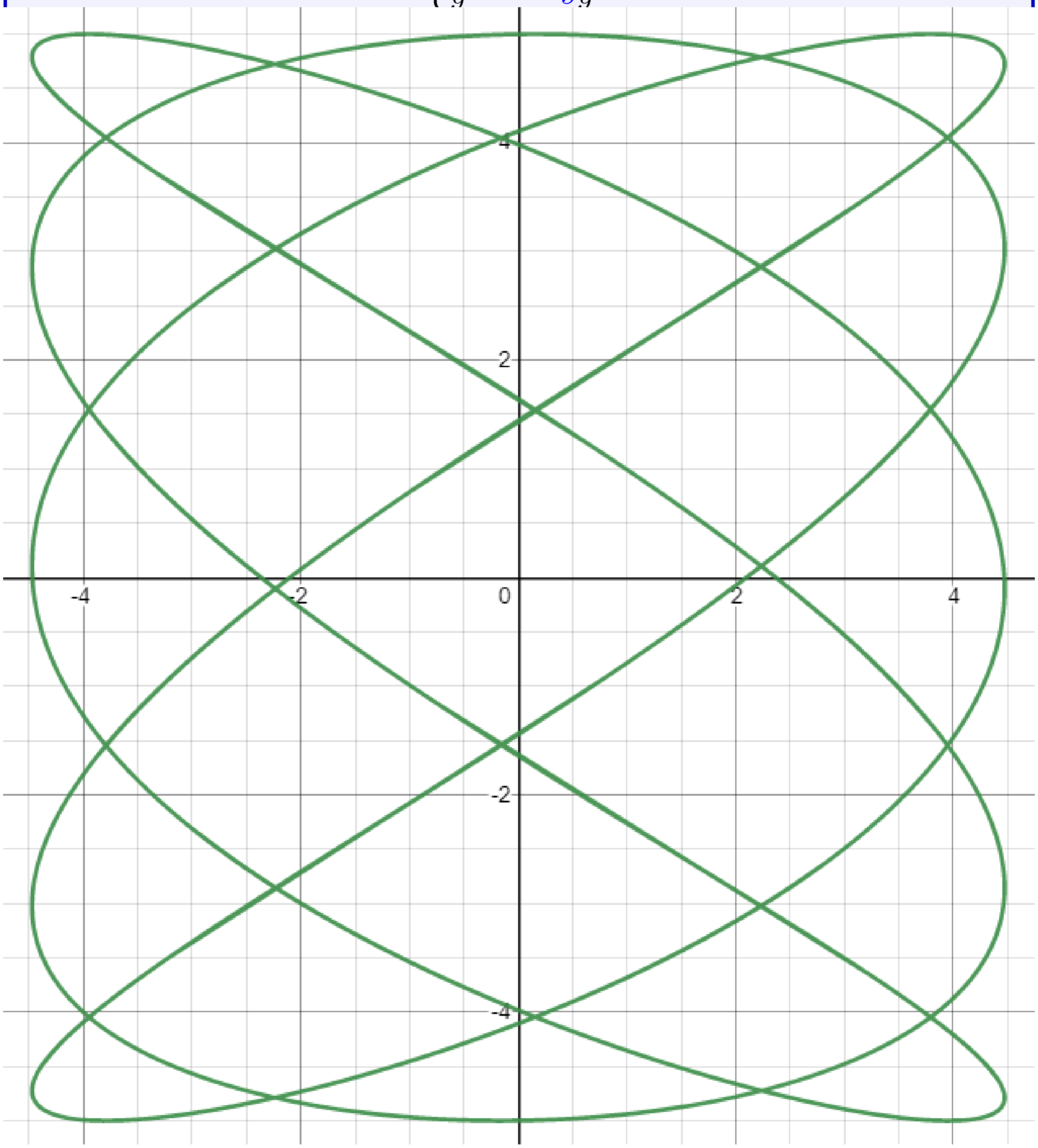
Example 6: (Coupled Harmonic Oscillator 2)

$$\begin{cases} x'' = -4x \\ y'' = -9y \end{cases}$$



Example 7: (Coupled Harmonic Oscillator 3)

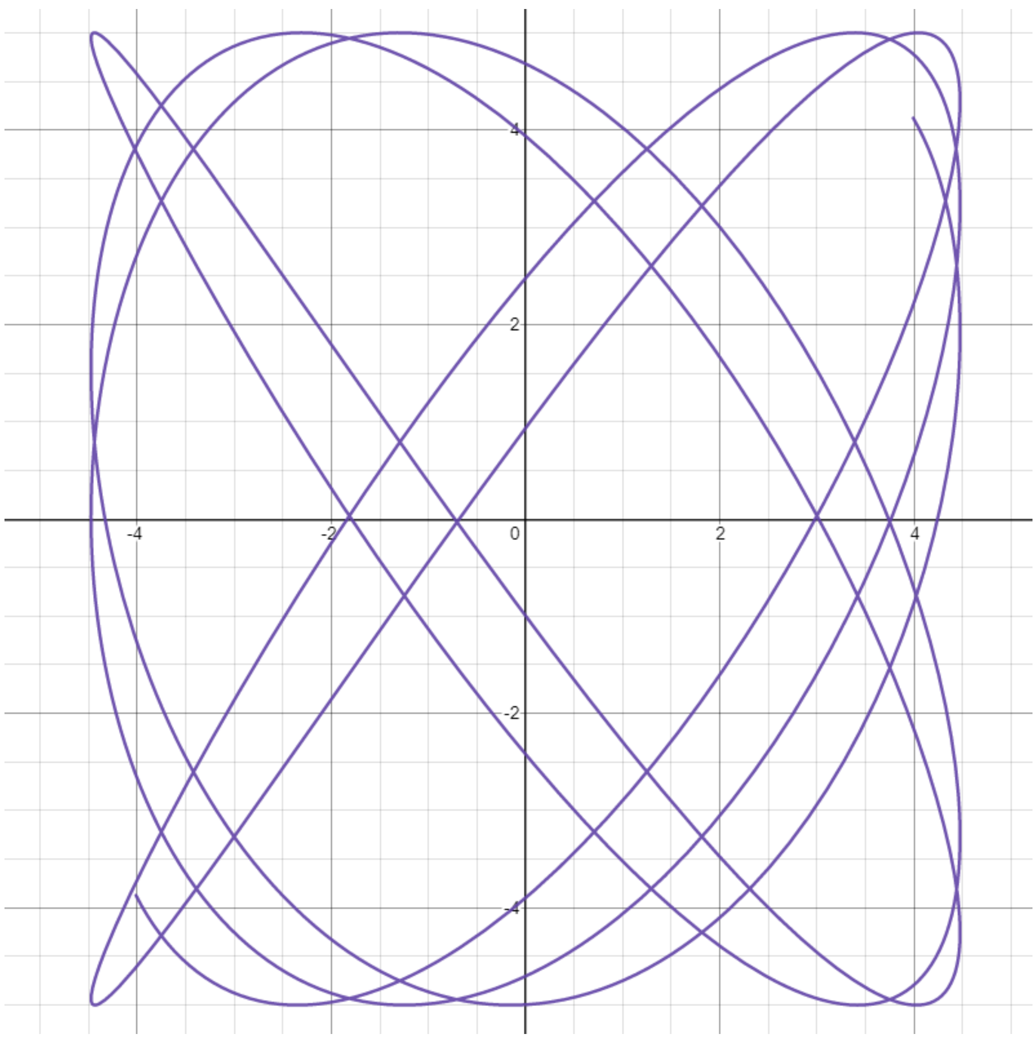
$$\begin{cases} x'' = -25x \\ y'' = -9y \end{cases}$$

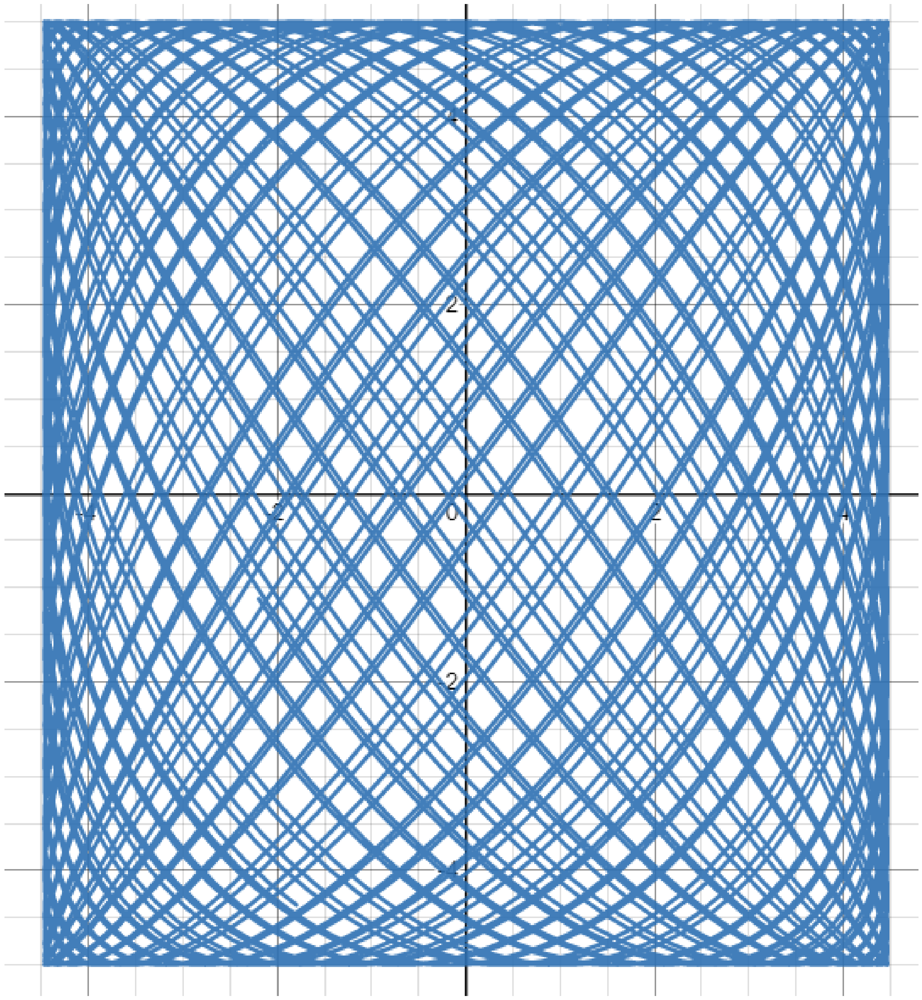


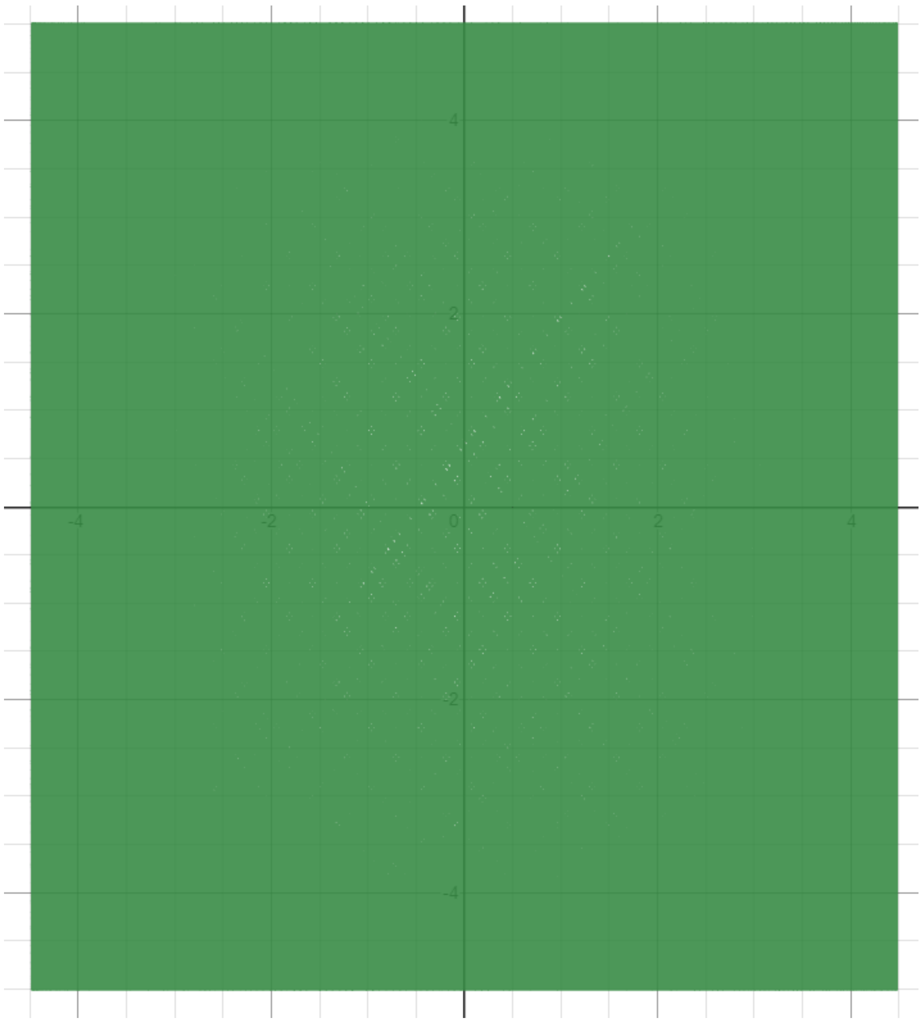
Note: Here the ratio of frequencies $\frac{5}{3}$ is rational. What if it's irrational?

Example 8: (Coupled Harmonic Oscillator 4)

$$\begin{cases} x'' = -2x \\ y'' = -3y \end{cases}$$







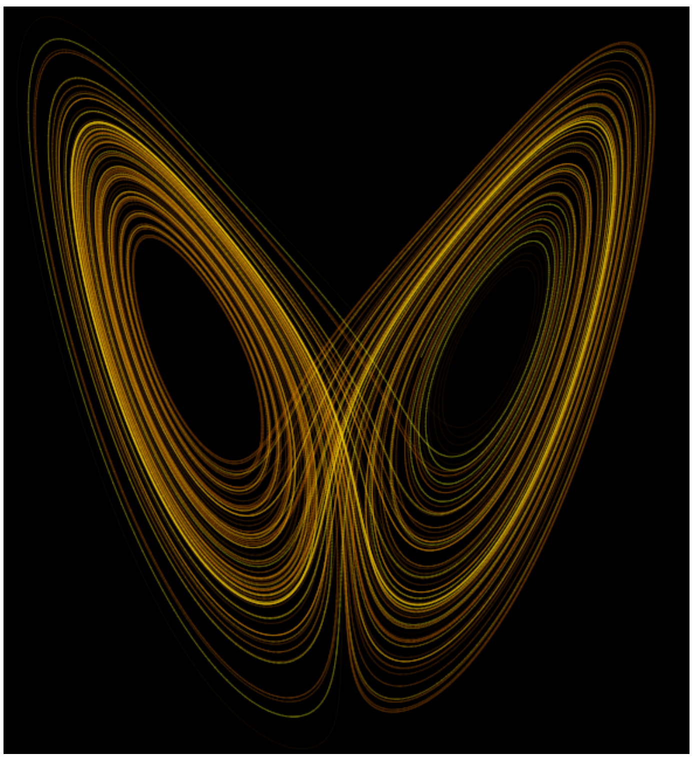
The solution fills up the square!!! In fact, even though before the solutions were periodic, here they are completely chaotic!

And in fact, this is the first one of **chaotic** ODE!

Example 9: (Lorenz Attractor)

$$\begin{cases} x' = -10x + 10y \\ y' = 28x - y - yz \\ z' = xy - \frac{8}{3}z \end{cases}$$

Initially used to predict the weather and not too crazy of a system at first sight, but the picture looks as follows (courtesy Wikipedia)



Here is an animation of the solution.

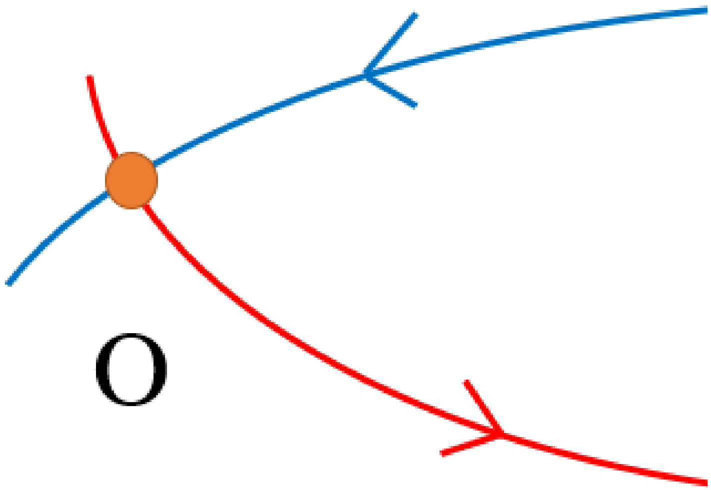
This is one of the first examples of a chaotic system, where the solution can't make up its mind!

Example 10:

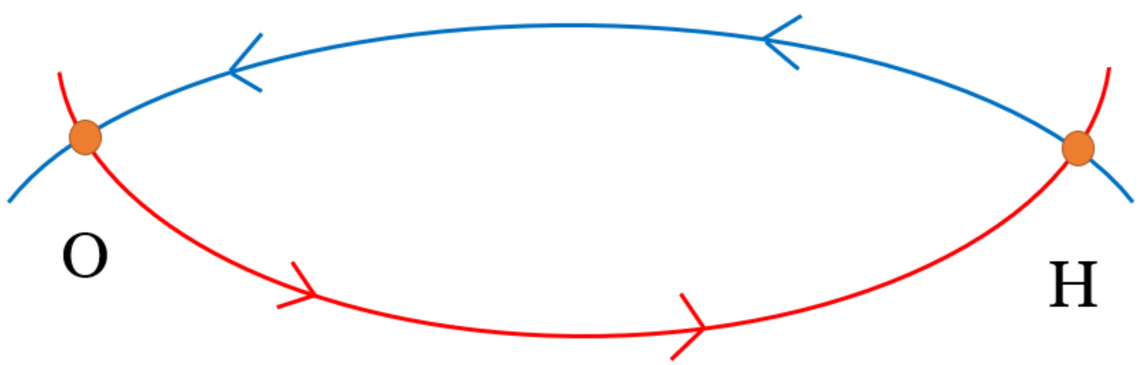
Homoclinic Point

This is part of a project I did as an undergrad in 2008.

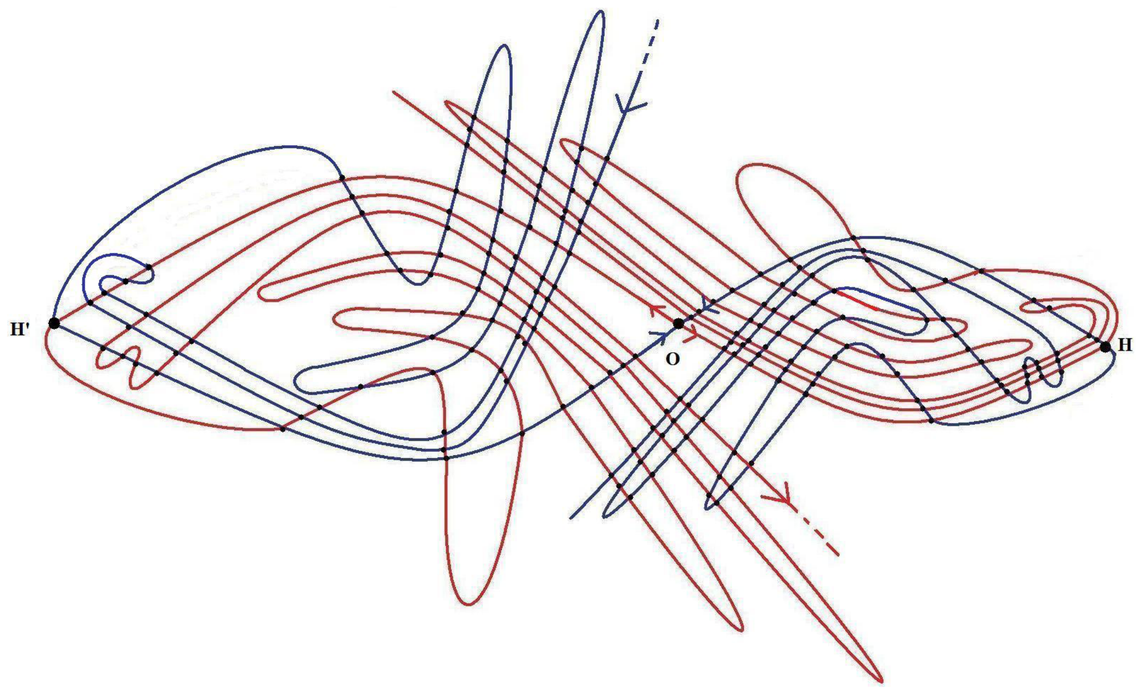
The premise is simple: Suppose you have an ODE such that there is a curve going into the origin O, and the a curve going out of it:



Moreover, suppose those two curves intersect again in a point H called a **homoclinic point**.



Then the whole picture looks like this:



So *one* homoclinic point implies *infinitely* many homoclinic points (each intersection is one) and the system is once again **chaotic**!