APMA 0350 – MIDTERM 1

1. *(5 points)* Solve the ODE and write your answer in explicit form

$$\begin{cases} t\left(y'\right)+2y=\dfrac{\cos(t)}{t} & (t>0) \\ y(\pi)=1 \end{cases}$$

2. *(5 points)* Solve the ODE and write your answer in explicit form

$$\begin{cases} y'=t\left(y^2\right)\left(1+t^2\right)^{-\frac{1}{2}} \\ y(0)=\dfrac{1}{3} \end{cases}$$

3. *(5 points)* Find the value of k for which this ODE is exact, and solve the ODE with that value of k. Write your answer in explicit form

$$\left(ye^{2xy}+x\right)dx+\left(kxe^{2xy}\right)dy=0$$

4. *(5 points)* Suppose you model the growth of bacteria with the following ODE

$$\begin{cases} \dfrac{dy}{dt}=(\sin(t))\,y \\ y(0)=y_0 \end{cases}$$

   Calculate the time it takes for the initial population to triple. Does your answer depend on y_0?

   Note: Please calculate the exact answer, no need to give an approximate answer.

5. *(5 points)* Use Euler's method with $N=2$ steps to find an approximate value of $y(5)$ where y solves

$$\begin{cases} y' = 2y - 1 \\ y(1) = 3 \end{cases}$$

Note: No need to actually solve the ODE

6. *(5 points)* Use auxiliary equations to solve

$$\begin{cases} y'' - 2y' + 5y = 0 \\ y\left(\frac{\pi}{2}\right) = 0 \\ y'\left(\frac{\pi}{2}\right) = -2 \end{cases}$$