## APMA 0350 - MIDTERM 2

| Name |  |
| :---: | :--- |
| Brown ID |  |
| Signature |  |

Note: Please use your Brown ID from your card, NOT the Banner ID Instructions: Welcome to your midterm. You have 50 minutes to take this exam, for a total of 25 points. No books, notes, calculators, or cellphones are allowed. You are allowed a standard $8.5 \times 11$ two-sided cheat sheet. Please put your answers in the boxes provided. Remember that you are not only graded on your final answer, but also on your work. If you need to continue your work on scratch paper, please check the box "Work on scratch paper"

Academic Honesty Statement: With the signature above, I certify that the exam was taken by the person named and without any form of assistance and acknowledge that any form of cheating results in an automatic F in the course, and will be further subject to disciplinary consequences, pursuant to the Brown University Academic Code.

| $f(t)$ | $\mathcal{L}\{f(t)\}$ |
| :---: | :---: |
| 1 | $\frac{1}{s}$ |
| $e^{a t}$ | $\frac{1}{s-a}$ |
| $t^{n}$ | $\frac{n!}{s^{n+1}}$ |
| $\sin (a t)$ | $\frac{a}{s^{2}+a^{2}}$ |
| $\cos (a t)$ | $\frac{s}{s^{2}+a^{2}}$ |
| $u_{c}(t)$ | $\frac{e^{-c s}}{s}$ |
| $u_{c}(t) f(t-c)$ | $e^{-c s} \mathcal{L}\{f(t)\}$ |
| $e^{a t} f(t)$ | $F(s-a)$ |
| $\delta(t-c)$ | $e^{-c s}$ |
| $y^{\prime}$ | $s \mathcal{L}\{y\}-y(0)$ |
| $y^{\prime \prime}$ | $s^{2} \mathcal{L}\{y\}-s y(0)-y^{\prime}(0)$ |

$$
(f \star g)(t)=\int_{0}^{t} f(t-\tau) g(\tau) d \tau
$$

1. (6 points, 2 points each) Guess the form of the particular sol of
(a) $y^{\prime \prime}-4 y^{\prime}+5 y=\left(t^{3}+3 t^{2}\right) e^{2 t}$
(b) $y^{\prime \prime}-4 y^{\prime}+5 y=(2 t+3) \cos (t)$
(c) $y^{\prime \prime}-4 y^{\prime}+5 y=t e^{2 t} \sin (t)$

Note: No justification required here
(a) $y_{p}=\square$
(b) $y_{p}=\square$
(c) $y_{p}=$
2. (5 points) Find a particular solution of

$$
t^{2} y^{\prime \prime}-6 t^{2} y^{\prime}+9 t^{2} y=e^{3 t}
$$

$$
y_{p}=
$$

3. (10 points) Solve the following ODE

$$
\left\{\begin{aligned}
y^{\prime \prime}-7 y^{\prime}+10 y & =6 \delta(t-5) \\
y(0) & =3 \\
y^{\prime}(0) & =0
\end{aligned}\right.
$$

$\square$Work on Scratch Paper
4. (4 points) Solve the following integral equation

$$
\phi(t)-\int_{0}^{t}(t-\tau) \phi(\tau) d \tau=1+t
$$

$$
\phi=
$$

