

## APMA 0350 – MIDTERM 2

**Note:** Don't forget to use the Laplace table on the course website.

1. (6 points) Find the eigenvalues and eigenfunctions of

$$\begin{cases} y'' = \lambda y \\ y(0) = 0 \\ y'(2\pi) = 0 \end{cases}$$

2. (6 points) Solve using undetermined coefficients

$$\begin{cases} y'' + 4y = 4 \sin(2t) \\ y(0) = 0 \\ y'(0) = 7 \end{cases}$$

3. (6 points) Use var of par to find a particular solution  $y_p$  of

$$t^2 (y'') + t (y') + \left(t^2 - \frac{1}{4}\right) y = t^{\frac{3}{2}}$$

**Note:** Assume that  $t^{-\frac{1}{2}} \cos(t)$  and  $t^{-\frac{1}{2}} \sin(t)$  solve the homogeneous equation. Simplify your final answer.

4. (6 points) Solve the following ODE

$$\begin{cases} y'' + 9y = (18t) \star \delta(t - 5) \\ y(0) = 0 \\ y'(0) = 1 \end{cases}$$

5. (6 points) Find a function  $f(t)$  whose Laplace transform is

$$\frac{(s + 1)e^{-4s}}{s^2 + 6s + 11}$$