APMA 0350 – FINAL EXAM

1. *(5 points)* Solve the following system $\mathbf{x}' = A\mathbf{x}$ and draw a phase portrait where

$$A = \begin{bmatrix} 6 & -1 \\ 5 & 4 \end{bmatrix}$$

2. *(5 points)* Use matrix exponentials to solve $\mathbf{x}' = A\mathbf{x}$ where

$$A = \begin{bmatrix} -6 & 4 \\ -1 & -2 \end{bmatrix} \quad \mathbf{x}(0) = \begin{bmatrix} 2 \\ -3 \end{bmatrix}$$

3. *(5 points, 1 point each)* Guess the form of the particular solution $\mathbf{x_p}$ to $\mathbf{x}' = A\mathbf{x} + \mathbf{f}$ where

$$A = \begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix}$$

   (a) $\mathbf{f} = \begin{bmatrix} e^{2t} \\ 3e^{2t} \end{bmatrix}$

   (b) $\mathbf{f} = \begin{bmatrix} 7te^{3t} \\ 4e^{-4t} \end{bmatrix}$

   (c) $\mathbf{f} = \begin{bmatrix} 2e^{5t} \\ e^{5t} \end{bmatrix}$

   (d) $\mathbf{f} = \begin{bmatrix} e^{5t}\cos(t) \\ e^{3t}\sin(2t) \end{bmatrix}$

   (e) $\mathbf{f} = \begin{bmatrix} 2e^{5t} \\ e^{-t} \end{bmatrix}$

4. *(5 points)* Use Var of Par to find a particular solution $\mathbf{x_p}$ to $\mathbf{x}' = A\mathbf{x} + \mathbf{f}$ where

$$A = \begin{bmatrix} 2 & -5 \\ 1 & -2 \end{bmatrix} \text{ and } \mathbf{f} = \begin{bmatrix} 5\sec(t) \\ 0 \end{bmatrix}$$

5. *(5 points)* Find and classify the equilibrium point(s) of

$$\begin{cases} x' = -x + 2xy \\ y' = y - x^2 - y^2 \end{cases}$$

6. *(5 points)* Let's model the population of elves between Chocolatetown and Vanillaville. Every day, we simultaneously have

- 30 elves exit Chocolatetown (not from Vanilaville)
- 20% of elves move from Chocolatetown to Vanillaville
- 50% of elves move from Vanillaville to Chocolatetown
- 10 elves enter Vanillaville (not to Chocolatetown)

Let $x(t)$ and $y(t)$ be the number of elves in Chocolatetown and Vanillaville respectively, where t is in days and let $\mathbf{x}(t) = \begin{bmatrix} x(t) \\ y(t) \end{bmatrix}$

Suppose our model is $\mathbf{x}' = A\mathbf{x} + \mathbf{f}$

Find A and $\mathbf{f}$ and include a diagram similar to the chemical tank problem. No justification required