# ANALYSIS – HOMEWORK 8

**Problem 1:** Show, directly, using the definition of the Darboux integral that

$$\int_0^1 x^2 dx = \frac{1}{3}$$

**Problem 2:** More generally, show that if $f$ is monotonic (increasing or decreasing) on $[a, b]$ then $f$ is Darboux integrable on $[a, b]$

**Problem 3:** Prove the Darboux Integrability Criterion: $f$ is Darboux integrable if and only if for all $\epsilon > 0$ there is a partition $P$ of $[a, b]$ such that $U_P(f) - L_P(f) < \epsilon$

**Problem 4:** Show that if $f$ is continuous on $[a, b]$ then $f$ is integrable on $[a, b]$

**Problem 5:** Show that if $f$ is integrable on $[a, b]$ then so is $f^2$ (Remember $f$ is bounded here). Show the result is false on $(0, 1)$ if $f$ is not bounded.

**Problem 6:** Prove the Fundamental Theorem of Calculus 2: If $f$ is $C^1$ then

$$\int_a^b f'(x) dx = f(b) - f(a)$$

**Hint:** Use the Mean-Value Theorem on each sub-piece

**Problem 7:** Prove the Fundamental Theorem of Calculus 1: If $f$ is continuous on $[a, b]$ and

$$F(x) = \int_a^x f(t)dt$$

Then $F'(x) = f(x)$

**Problem 8:** This is just pure calculus, but fun! Calculate

$$\int x \tan^{-1}(x)dx$$

But instead of choosing the antiderivative $\frac{x^2}{2}$ do it by choosing the antiderivative $\frac{x^2+1}{2}$, since adding constants doesn't matter!