APMA 0350 - FINAL EXAM

Name	
Brown ID	
Signature	

1. (5 points) Solve the ODE and write your answer in explicit form

$$\begin{cases} y' = 4t \left(y^2 + 1 \right) \\ y(0) = 1 \end{cases}$$

y =

2. (5 points) Solve the following exact ODE and write your answer in explicit form. Don't forget to check for exactness

$$\begin{cases} \left(\frac{2x}{y}+4\right)dx - \left(\frac{x^2}{y^2}\right)dy = 0\\ y(2) = 1 \end{cases}$$

g = 1

3. (5 points) Find the eigenvalues and eigenfunctions of

$$\begin{cases} y'' = \lambda y \\ y'(0) = 0 \\ y'(4) = 0 \end{cases}$$

Eigenvalues
Eigenfunctions

4. (5 points) Solve using undetermined coefficients

$$\begin{cases} y'' + 4y = 12t^2 + 20t + 30 \\ y(0) = 8 \\ y'(0) = -3 \end{cases}$$

y =

5. (5 points) Use the Laplace transform to solve the following ODE

$$\begin{cases} y'' + 2y' + 2y = f(t) \\ y(0) = 0 \\ y'(0) = 0 \end{cases} \text{ where } f(t) = \begin{cases} 0 & \text{if } 0 \le t < \pi \\ 2 & \text{if } \pi \le t < 2\pi \\ 0 & \text{if } t \ge 2\pi \end{cases}$$

y =	
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6. (5 points) Solve the following Integral Equation

$$\phi(t) + \int_0^t (t - \tau) \,\phi(\tau) d\tau = 1$$

$\phi(t) =$

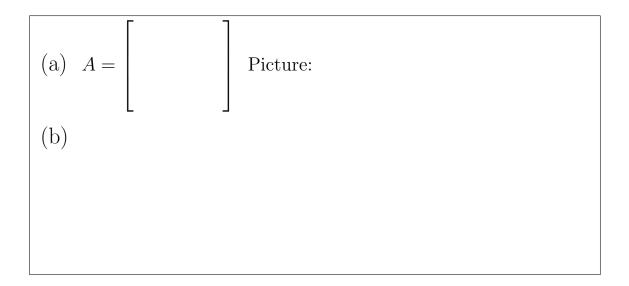
- 7. (5 = 3 + 2 points) Suppose you're modeling the population of bunnies between two cities Bunlandia and Doeville. Every day, we simultaneously have
 - $\bullet~60\%$ of bunnies from Bunlandia move to Doeville
 - \bullet 30% of bunnies from Doeville move to Bunlandia

Assume rabbits can <u>only</u> move between those two cities and that there are no foreign rabbits entering or exiting the two cities.

Let x(t) and y(t) be the number of bunnies in Bunlandia and Doeville respectively, where t is in days.

- (a) Set up an ODE model of the form $\mathbf{x}' = A\mathbf{x}$ where $\mathbf{x}(t) = \begin{bmatrix} x(t) \\ y(t) \end{bmatrix}$ but do **NOT** solve it. Include a picture like in the chemical tank model. No justification required.
- (b) Use your model in (a) to (mathematically) show that the total population x(t) + y(t) is constant

Hint: How do you show in that a function is constant?



8. (5 points) Use matrix exponentials to solve $\mathbf{x}' = A\mathbf{x}$ where

$$A = \begin{bmatrix} 5 & -1 \\ 3 & 1 \end{bmatrix} \quad \mathbf{x}(0) = \begin{bmatrix} 4 \\ 8 \end{bmatrix}$$

NO phase portrait required. Write your final answer using terms of the form $e^{at} \begin{bmatrix} b \\ c \end{bmatrix}$ where a, b, c are integers.

$\mathbf{x}(t) =$	

9. (5 points) Use <u>Variation of Parameters</u> to find a particular solution $\mathbf{x}_{\mathbf{p}}$ of $\mathbf{x}' = A\mathbf{x} + \mathbf{f}$ where

$$A = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \text{ and } \mathbf{f} = \begin{bmatrix} 6e^{4t} \\ 4e^{3t} \end{bmatrix}$$

Do **NOT** use undetermined coefficients! Write your answer using terms of the form $e^{at} \times$ Some vector

$\mathbf{x}_{\mathbf{p}} =$	

10. (5 points) Find and classify the equilibrium point(s) of

$$\begin{cases} x' = (y - x) (1 - x - y) \\ y' = x (2 + y) \end{cases}$$

Equilibrium Point	Classification