APMA 0350 – FINAL EXAM

Name	
Brown ID	
Signature	

1. *(5 points)* Solve the ODE and write your answer in explicit form

$$\begin{cases} y' = 4t\left(y^2+1\right) \\ y(0) = 1 \end{cases}$$

$y =$	

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2. *(5 points)* Solve the following exact ODE and write your answer in explicit form. Don't forget to check for exactness

$$\begin{cases} \left(\frac{2x}{y} + 4\right) dx - \left(\frac{x^2}{y^2}\right) dy = 0 \\ y(2) = 1 \end{cases}$$

$y =$	

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3. *(5 points)* Find the eigenvalues and eigenfunctions of

$$\begin{cases} y'' = \lambda y \\ y'(0) = 0 \\ y'(4) = 0 \end{cases}$$

Eigenvalues	
Eigenfunctions	

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4. *(5 points)* Solve using undetermined coefficients

$$\begin{cases} y'' + 4y = 12t^2 + 20t + 30 \\ y(0) = 8 \\ y'(0) = -3 \end{cases}$$

$y =$	

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5. *(5 points)* Use the Laplace transform to solve the following ODE

$$\begin{cases} y'' + 2y' + 2y = f(t) \\ y(0) = 0 \\ y'(0) = 0 \end{cases} \quad \text{where } f(t) = \begin{cases} 0 & \text{if } 0 \leq t < \pi \\ 2 & \text{if } \pi \leq t < 2\pi \\ 0 & \text{if } t \geq 2\pi \end{cases}$$

$y =$

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6. *(5 points)* Solve the following Integral Equation

$$\phi(t) + \int_0^t (t - \tau)\,\phi(\tau)d\tau = 1$$

$\phi(t) =$	

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7. *(5 = 3 + 2 points)* Suppose you're modeling the population of bunnies between two cities Bunlandia and Doeville. Every day, we simultaneously have

- 60% of bunnies from Bunlandia move to Doeville
- 30% of bunnies from Doeville move to Bunlandia

Assume rabbits can only move between those two cities and that there are no foreign rabbits entering or exiting the two cities.

Let $x(t)$ and $y(t)$ be the number of bunnies in Bunlandia and Doeville respectively, where t is in days.

(a) Set up an ODE model of the form $\mathbf{x}' = A\mathbf{x}$ where $\mathbf{x}(t) = \begin{bmatrix} x(t) \\ y(t) \end{bmatrix}$ but do **NOT** solve it. Include a picture like in the chemical tank model. No justification required.

(b) Use your model in (a) to (mathematically) show that the total population $x(t) + y(t)$ is constant

Hint: How do you show in that a function is constant?

(a) $A = \begin{bmatrix} \quad & \quad \\ \quad & \quad \end{bmatrix}$ Picture:

(b)

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8. *(5 points)* Use **matrix exponentials** to solve $\mathbf{x}' = A\mathbf{x}$ where

$$A = \begin{bmatrix} 5 & -1 \\ 3 & 1 \end{bmatrix} \quad \mathbf{x}(0) = \begin{bmatrix} 4 \\ 8 \end{bmatrix}$$

NO phase portrait required. Write your final answer using terms of the form $e^{at}\begin{bmatrix} b \\ c \end{bmatrix}$ where a, b, c are integers.

$\mathbf{x}(t) =$	

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9. *(5 points)* Use **Variation of Parameters** to find a particular solution $\mathbf{x_p}$ of $\mathbf{x}' = A\mathbf{x} + \mathbf{f}$ where

$$A = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \text{ and } \mathbf{f} = \begin{bmatrix} 6e^{4t} \\ 4e^{3t} \end{bmatrix}$$

Do **NOT** use undetermined coefficients! Write your answer using terms of the form $e^{at} \times$ Some vector

$\mathbf{x_p} =$	

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10. *(5 points)* Find and classify the equilibrium point(s) of

$$\begin{cases} x' = (y - x)(1 - x - y) \\ y' = x(2 + y) \end{cases}$$

Equilibrium Point	Classification

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