APMA 1650 Hw 1 Solutions

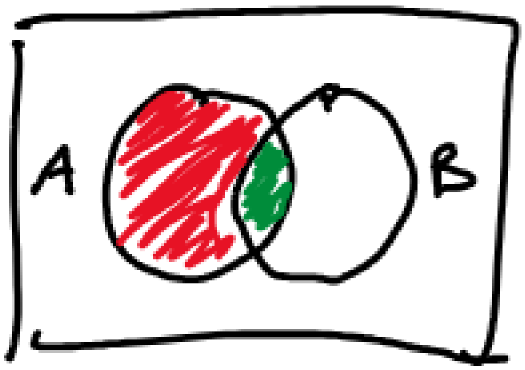
September 18, 2023

Section 2.3

2.4

If A and B are two sets, draw Venn diagrams to verify the following:

$A = (A \cap B) \cup (A \cap \overline{B})$



If $B \subset A$, then $A = B \cup (A \cap \overline{B})$

2.6

Suppose two dice are tossed and the numbers on the upper faces are observed. Let S denote the set of all possible pairs that can be observed. [These pairs can be listed, for example, by letting $(2, 3)$ denote that a 2 was observed on the first die and a 3 on the second.] Define the following subsets of S:

- A: The number on the second die is even.
- B: The sum of the two numbers is even.
- C: At least one number in the pair is odd.

List the points in A, $\overline{C}$, $A \cap B$, $A \cap \overline{B}$, $\overline{A} \cup B$, and $\overline{A} \cap C$.

- $A = \{(1,2),(2,2),(3,2),(4,2),(5,2),(6,2),$
  $(1,4),(2,4),(3,4),(4,4),(5,4),(6,4),$
  $(1,6),(2,6),(3,6),(4,6),(5,6),(6,6)\}$
- $\overline{C} = \{(2,2),(2,4),(2,6),(4,2),$
  $(4,4),(4,6),(6,2),(6,4),(6,6)\}$
- $A \cap B = \{(2,2),(4,2),(6,2),$
  $(2,4),(4,4),(6,4),(2,6),(4,6),(6,6)\}$
- $A \cap \overline{B} = \{(1,2),(3,2),(5,2),$
  $(1,4),(3,4),(5,4),(1,6),(3,6),(5,6)\}$
- $\overline{A} \cup B = \{(1,1),(1,3),(1,5),$
  $(2,1),(2,2),(2,3),(2,4),(2,5),(2,6),$
  $(3,1),(3,3),(3,5),(4,1),(4,2),$
  $(4,3),(4,4),(4,5),(4,6),(5,1),$
  $(5,3),(5,5),(6,1),(6,2),(6,3),(6,4),(6,5),(6,6)\}$
- $\overline{A} \cap C = \overline{A}$

2.8

From a survey of 60 students attending a university, it was found that 9 were living off campus, 36 were undergraduates, and 3 were undergraduates living off campus. Find the number of these students who were

- undergraduates, were living off campus, or both
- undergraduates living on campus
- graduate students living on campus

	On-Campus	**Off-Campus**	**Total**
Undergraduate	33	3	36
Graduate	18	6	24
Total	51	9	60

- $36 + 9 - 3 = 42$
- 33
- 18

AP1

Give an example of three sets A, B, C such that $A \cap B \cap C = \emptyset$ but A, B, C are not pairwise disjoint.
Let

- $A = \{1, 2\}$

- $B = \{2, 3\}$
- $C = \{3, 1\}$

Then

- $A \cap B = \{2\}$
- $B \cap C = \{1\}$
- $C \cap A = \{3\}$

But $A \cap B \cap C = \emptyset$.

Now prove in general that if A, B, C are pairwise disjoint, then $A \cap B \cap C = \emptyset$.

Because A, B, C are pairwise disjoint, $A \cap B = B \cap C = C \cap A = \emptyset$. Assume that $A \cap B \cap C \neq \emptyset$. Then there exists $x \in A \cap B \cap C$. Then $x \in A$, $x \in B$, and $x \in C$. But then $x \in A \cap B$, a contradiction as $A \cap B = \emptyset$.

Section 2.4

2.15

An oil prospecting firm hits oil or gas on 10% of its drillings. If the firm drills two wells, the four possible simple events and three of their associated probabilities are as given in the accompanying table. Find the probability that the company will hit oil or gas

a

on the first drilling and miss on the second.

Since the events are Mutually Exclusive:

$E_1 + E_2 + E_3 + E_4 = 1 \implies .01 + E_2 + .09 + .81 = 1 \implies E_2 + .91 = 1 \implies E_2 = .09$

b

on at least one of the two drillings.

$1 - E_4 = 1 - .81 = .19$

2.16

a

$\frac{1}{3}$

b

$\frac{1}{3} + \frac{1}{15} = \frac{6}{15} = \frac{2}{5}$

c

$\frac{1}{3} + \frac{1}{16} = \frac{16}{48} + \frac{3}{48} = \frac{19}{48}$

d

$1 - (P(A) + P(O)) = 1 - (\frac{2}{5} + \frac{19}{48}) = 1 - \frac{191}{240} = \frac{49}{240}$

2.18

a

$\Omega = \{(H,H),(H,T),(T,H),(T,T)\}$

b

$P((H,H)) = P((T,T)) = P((H,T)) = P((T,H)) = .25$
Yes, if the coin is fair

c

$A = \{(H,T),(T,H)\}$
$B = \{(H,T),(T,H),(H,H)\} = A \cup \{(H,H)\}$

d

- $P(A) = .5$
- $P(B) = .75$
- $P(A \cap B = A) = .5$
- $P(A \cup B = B) = .75$
- $P(\bar{A} \cup B = \Omega) = 1$

AP2

Since A and A^c are disjoint, by axiom 5:

$$P(A^c \cup A) = P(A^c) + P(A)$$

Now by axiom 3, $P(S) = 1$ so:

$$1 = P(S = A^c \cup A) = P(A^c) + P(A) \implies P(A^c) = 1 - P(A)$$