

APMA 1650 – HOMEWORK 2

Problem 1: Suppose you flip a fair coin 50 times.

- (a) What is the size of the sample space for this experiment, i.e. how many outcomes are possible?
- (b) What is the probability that you flip exactly 10 heads?
- (c) What is the probability that you flip at least 10 heads?
- (d) What is the probability that you never flip two heads in a row or two tails in a row?

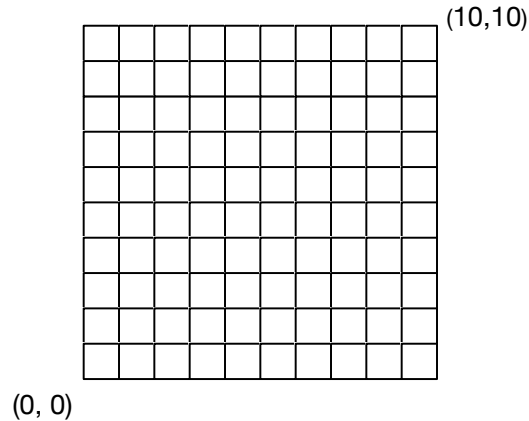
Note: You may leave the answers in terms of binomial coefficients and exponents

Problem 2: You enlist a friend from materials science to construct a very special unfair six-sided die. The die looks like a standard die, i.e. it is cubical and has the numbers 1, 2, 3, 4, 5, and 6 on its faces. On this die, the probability of rolling any number is directly proportional to that number. For example, you are twice as likely to roll a 6 than a 3.

- (a) What is the probability of rolling each of the six numbers?
- (b) What is the probability of rolling an odd number?

Problem 3: Consider the grid of points shown below. You start at $(0, 0)$ and take one step either up or to the right with each move. You keep moving until you reach $(10, 10)$. You can never leave the grid. For example, if you reach the point $(4, 10)$, you can only move to the right from that point onwards. Assume that each possible path is equally likely.

- (a) What is the size of the sample space, i.e. how many possible paths are there from $(0, 0)$ to $(10, 10)$?
- (b) What is the probability that a path passes through $(5, 5)$?



Note: You may leave the answers in terms of binomial coefficients and exponents.

Problem 4: Powerball is an American lottery game offered by 44 states. To play the game, you select 5 distinct numbers from a set of 69 white balls (numbered 1 - 69) and one number from a set of 26 red Powerballs (numbered 1 - 26). In each drawing, five white balls and one red Powerball are selected. The order of the white balls does not matter.

- (a) You win the jackpot if you match all 5 white balls and the Powerball. What is the probability that you win the jackpot?
- (b) If you match all 5 white balls but do not match the Powerball, you win \$1,000,000. What is the probability that this occurs?
- (c) If you match the Powerball but do not match *any* of the white balls, you win \$4. What is the probability that this occurs?

- (d) If you match *exactly* 3 white balls (so you don't match the other two balls) and the Powerball, you win \$100. What is the probability that this occurs?

Problem 5: How many distinct arrangements are there of the word HULLABALLOO?

Problem 6: In this problem, write your answer in terms of binomial coefficients and also, using a calculator, calculate an approximate value (up to 2 decimal places). Suppose you're dealt a 5 card poker hand (out of 52 cards). What is the probability of drawing

- (a) Royal Flush: 10 J Q K A of the same suit
- (b) Straight flush (excluding royal flush): Any consecutive order of cards of the same suit, like 45678
- (c) Four of a kind, like 3333A
- (d) Flush (excluding royal and straight flush): Five cards of the same suit
- (e) Straight (excluding royal flush and straight flush): Any consecutive order of cards, not necessarily of the same suit, like 78910J
- (f) Three of a kind, like 444JK
- (g) Two pairs, like 33558
- (h) One pair, like 224JK

Note: Feel free to check out this video¹ on how to do this:

Video: The Math of Poker

¹In that video, the probability of Straight is slightly incorrect, the 11 should be 10