

## APMA 1650 – HOMEWORK 3

**Problem 1:** You have a bag containing  $r$  red marbles and  $g$  green marbles, where  $r$  and  $g$  are positive integers. You draw marbles from the bag, but there is a twist! Each time you draw a marble from the bag, you replace it **and** add another marble of the same color. Assume you never run out of marbles to add to the bag.

- (a) What is the probability of getting a red marble on the first draw?
- (b) What is the probability of getting a red marble on the second draw?
- (c) Make a guess for the probability of getting a red marble on the  $n$ th draw.

**Problem 2:** You decide to use an ATM to get cash before leaving for vacation. Unfortunately, when you insert your ATM card, you realize you have forgotten your PIN (4-digit password). On the bright side, you know that it is one of 10 PINs which you use. You decide to try your PINs uniformly at random, never retrying a PIN you have already tried. The ATM will disable your card after 3 incorrect attempts. What is the probability that you get your PIN correct before your card is disabled?

**Problem 3:** The eight Ivy-league schools (Brown, Columbia, Cornell, Dartmouth, Harvard, Penn, Princeton, and Yale) are having a badminton tournament. In the first round, the teams are paired off uniformly at random. How many possible outcomes are there for the first round? An “outcome” specifies both the team pairings and the winners. Here is one such outcome:

- Brown beats Yale
- Dartmouth beats Harvard
- Princeton beats Penn
- Columbia beats Cornell

**Problem 4:** You are the quality control manager for the Acme Widget Company. You have three factories which produce widgets. The table below gives the fraction of the total widget production allotted to each factory as well as the fraction of defective widgets produced by each factory.

Factory	Fraction of total production	Fraction of defective widgets
A	0.20	0.020
B	0.30	0.010
C	0.50	0.005

You test a randomly-selected widget and find that it is defective. What is the probability that it came from Factory A? (Find an approximate value up to two decimal places)

**Problem 5:** Suppose your 5-card poker hand contains at least 2 aces, what is the probability that it contains all 4 aces? You may leave your answer in terms of binomial coefficients.