APMA 1650 – HOMEWORK 4

Problem 1: You have m red marbles which you will put uniformly at random into n boxes. What is the expected number of boxes which are empty after all m marbles have been distributed.

Hint: Use the method of indicators

Problem 2: Two people are playing a game. They take turns rolling a standard, fair six-sided die. The game ends when one player rolls a 6. The player who rolls the 6 is the winner of the game. A "round" of the game is defined as a single die roll.

(a) What is the probability that the player who goes first wins?

(b) What is the expected value and variance for the number of rounds in the game?

(c) What is the probability that the game lasts at least 4 rounds?

(d) Given the game has lasted four rounds, what is the probability that the game lasts at least 8 rounds?

Problem 3: A single cell will die with probability p or split into two cells with probability $1 - p$, producing a second generation of cells. Each cell in the second generation (if there are any) will die or split into two with the same probabilities as the initial cell. You start with a single cell.

(a) Find the probability mass function (pmf) for the number of cells in the third generation. It might be useful to draw a probability tree **(TURN PAGE)**

(b) What is the expected value of the number of cells in the third generation?

Problem 4: A particular flight on Peyamerican Airlines can only fit 200 people, but tickets were sold to 205 people. Suppose each ticket holder has a 0.05 probabiltiy of not showing up for the flight.

(a) What is the probability that the flight will be overbooked? Give an approximate numerical answer for this.

(b) What is the expected number of people that show up for the flight?

(c) What is the variance of the number of people who show up for the flight?

Problem 5: You reprise your role as the quality control manager for the Acme Widget Company. You have found that in every box of 100 widgets there is on average 1 defective widget.

(a) Model this problem with an appropriate probability distribution. What is the probability that a box of 100 widgets contains 2 or fewer defective widgets? Also give an approximate numerical answer for this.

(b) **(optional)** Approximating this with a Poisson distribution, find the probability that you have 2 or fewer defective widgets.

(c) **(optional)** Evaluating both part (a) and part (b) numerically using Wolfram Alpha or your favorite software package, what is the relative error |True value - Approximate value| / True value for your Poisson approximation?