APMA 0350 – MIDTERM 1 – STUDY GUIDE

This is the study guide for the exam, and is just meant to be a *guide* to help you study, just so we're on the same place in terms of expectations. Think of it more as a course summary rather than "this is exactly the questions I'm going to ask you on the exam."

There will be **NO** coding on the exam, but there might be proofs, like on the homework

Useful trig identities to know:

(1) $\sin^2(x) + \cos^2(x) = 1$

(2) $1 + \tan^2(x) = \sec^2(x)$

(3) $\cos(-x) = \cos(x), \sin(-x) = -\sin(x)$

(4) $\cos(2x) = \cos^2(x) - \sin^2(x)$, $\sin(2x) = 2\sin(x)\cos(x)$

(5) $\cos^2(x) = \frac{1}{2} + \frac{1}{2}\cos(2x)$, $\sin^2(x) = \frac{1}{2} - \frac{1}{2}\cos(2x)$

Useful integration techniques to know:

(1) Integrals of $\sin, \cos, \tan, \sec$

(2) $\int \frac{1}{x^2+1} dx = \tan^{-1}(x) + C$

(3) $u-$substitution

(4) Integration by parts, but I'll only ask you about easy cases like $\int xe^x dx$

(5) $\int \cos^2(x)dx, \int \sin^2(x)dx$ Check out this video in case you forgot

(6) You only need partial fractions for the logistic equation

1. Introduction (Lectures 1 – 3)

- Solve the most basic differential equation

$$\begin{cases} y' = ky \\ y(0) = y_0 \end{cases}$$

  You can go directly to $y = y(0)e^{kt}$ do not use separation of variables for this

- Show that there are no other solutions to this ODE, by calculating $\left(ye^{-kt}\right)'$ Also look at the homework problem where you calculated $\left(e^{-2t}y\right)''$

- Check if a function solves an ODE. This just means to plug in the function into the ODE and see if you get an identity.

- Find the order of an ODE and whether it is linear/nonlinear and homogeneous/inhomogeneous

- Plot the direction field of a differential equation. To be honest, this is best done with a computer. Ignore the part about the dfield app

- Find the equilibrium solutions of an autonomous ODE and draw a bifurcation diagram (the table with arrows) and use that to determine the stability of equilibria.

2. Existence-Uniqueness (Lecture 3–4)

- Study the two crazy examples of non-existence/non-uniqueness
- Apply the Existence/Uniqueness Theorem to show that there is a unique solution to an ODE, like on the homework.

3. Separable ODE (Lecture4– 5)

- Solve an ODE using separation of variables
- This works well if you can put all the y's on one side and all the x's (or t's) on the other side
- I would tell you whether to leave your solution in implicit form or write it in explicit form
- **Careful of hidden solutions** For example, when you divide by y, you have to check if $y = 0$ is a solution. Sometimes it is, sometimes it isn't. Or if you divide by $y^2 - 1$, you have to check if $y = \pm 1$ is a solution.
- Solve the logistic differential equation $y' = Py\left(1 - \frac{y}{M}\right)$ with $y(0) = y_0$.
- In order to save you some time, you do **NOT** have to show the fact that $\frac{20}{y(20-y)} = \frac{1}{y} + \frac{1}{20-y}$

4. Integrating Factors (Lecture 5–6)

- Solve an ODE using **integrating factors**, both in the case $y'+ay$, where you multiply by e^{at} and $y'+Py$ where you multiply by $e^{\int P}$

- Make sure the coefficient of y' is 1 here, people usually lose points on that
- In general, integrating factors work if you can put your equation in the form $y' + Py =$ something

5. Applications (Lecture 6–7)

- All the application problems are fair game, including
  - Falling Object
  - Savings Model
  - Chemical Reactions
  - The rabbit problem on the homework
  - Newton's Law of Cooling on the homework
  - That water/pollutant problem on the homework
  - Bunny vs Foxes (end of the lecture notes)
- You do **NOT** need to memorize any physics equations, except for Newton's Second Law ($F = ma$), any other equations would be provided.
- I could ask you to derive some of those equations, using the trick with $h =$ small change in time and calculating for example $W(t + h) = W(t) +$ Change
- For the chemical tank problem, really think in terms of "what is going in" vs "what is going out," that should help you setting up the equations. Also the units are useful, usually you want something in kg/min

6. Exact Equations (Lecture 8)

- Put a differential equation in the form $Pdx + Qdy = 0$
- Solve exact ODE
- Of course this only works when $P_y = Q_x$ (PeYam = QuiXotic)
- Don't forget to check that your ODE is exact, otherwise you lose points
- Sometimes you can multiply an inexact equation by an integrating factor to make it exact, but in that case I would explicitly give you the integrating factor

7. Euler's Method (Lectures 9–10)

- Apply Euler's method with a small number of steps, like $N = 2$ or $N = 3$ steps. I'll make the algebra as reasonable as possible, since calculators are not allowed.
- Of course, know the formula for Euler's Method
- Remember that there is no coding on the exam
- You don't need to know the sections on Error and Problems with Euler

8. Second-Order ODE (Lectures 10 – 12)

- Use the **differential operators method** to solve a second-order ODE. That's the one where you use $Dy = y'$
- Solve a second-order ODE using auxiliary equations:

- ▶ Distinct Roots
- ▶ Repeated Roots
- ▶ Complex Roots

- Determine where a solution attains a max or a min (see lecture)
- I could ask you about higher order differential equations, but in that case would give you the roots beforehand