APMA 0350 – MIDTERM 1

1. *(5 points)* Solve the ODE and write your answer in explicit form

$$\begin{cases} t\,(y') = 3y + t^5 & t > 0 \\ y(1) = 5/2 \end{cases}$$

$y =$	

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2. *(5 points)* Solve the ODE and write your ans in implicit form

$$\begin{cases} \dfrac{dy}{dx} = \dfrac{\sin(x) - 2xy}{x^2 - \cos(y)} \\ y\left(\dfrac{\pi}{2}\right) = 0 \end{cases}$$

Answer:	

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3. *(5 points)* Solve the second-order ODE

$$\begin{cases} y'' + 4y' + 20y = 0 \\ y(0) = 2 \\ y'(0) = -8 \end{cases}$$

Note: You can do this directly, without differential operators

$y =$	

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4. *(5 points)* Find the equilibrium solutions of the following ODE and classify them as stable/unstable/bistable. You do **NOT** need to draw sample solutions

$$y' = -3\,(y+2)^3\,y\,(y-2)^2\,(y-4)$$

Equilibrium Sol	Classification

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5. *(5 points)* Here is another model for a rabbit population:

- The initial population is 100 rabbits
- Every rabbit has 3 offspring per day
- 15 rabbits per day are eaten by the local bird population

Find a differential equation for the population $P(t)$ of rabbits at time t (where t is in days) and solve it.

Note: You don't need to justify how you obtained your ODE, but you need to justify how you solved it.

ODE	
$P(t) =$	

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