APMA 1650 – HOMEWORK 6

Problem 1: You are the quality control manager for the Acme Widget Company. You have just launched a new line of MiniWidgets. Your MiniWidgets are produced by a MiniWidget machine. The MiniWidgets produced by the machine have masses which are normally distributed with a standard deviation of 0.2 grams. The machine can be adjusted so that the MiniWidgets it produces have an average mass of μ grams. What setting for μ should you use so that the masses of the MiniWidgets will exceed 10 grams at most 2% of the time?

Problem 2: Let X_1 and X_2 be two **independent** geometric random variables, both with parameter p. Find a nice, closed-form formula for

$$P(X_1 = i | X_1 + X_2 = n)$$

Your formula should not involve a sum.

Hint: Use the definition of conditional probability and notice $(X_1 = i)$ and $(X_1 + X_2 = n)$ implies $X_2 = n - i$

Problem 3: Suppose that the random variables Y_1 and Y_2 have joint density function:

$$f(y_1, y_2) = \begin{cases} y_1 + y_2 & 0 \leq y_1, y_2 \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

(a) Show this is a valid joint probability density.

(b) Find the marginal densities for Y_1 and Y_2 **(TURN PAGE)**

(c) Find the probability that $(Y_1 < 1/2)$ and $(Y_2 > 1/2)$.

(d) Find the conditional density for Y_1 given $(Y_2 = y_2)$.

(e) Find the probability that $(Y_1 < 1/2)$ given $(Y_2 = 1/2)$.

Problem 4: Suppose that the random variables Y_1 and Y_2 have joint density function:

$$f(y_1, y_2) = \begin{cases} c(1 - y_2) & 0 \leq y_1 \leq y_2 \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

(a) Find the value of c that makes this a valid joint probability density.

(b) Find the marginal densities for Y_1 and Y_2.

(c) Find the conditional density for Y_1 given $(Y_2 = y_2)$

(d) Find the expected values $E(Y_1)$ and $E(Y_2)$.

Problem 5: Suppose that the random variables Y_1 and Y_2 have joint density function:

$$f(y_1, y_2) = \begin{cases} e^{-(y_1+y_2)} & y_1 > 0, y_2 > 0 \\ 0 & \text{otherwise} \end{cases}$$

(a) Find the marginal densities for Y_1 and Y_2. What kind of random variables are Y_1 and Y_2?

(b) Find the conditional density of Y_1 given that $(Y_2 = y_2)$ for $y_2 > 0$

(c) Are Y_1 and Y_2 independent? Justify your answer.