APMA 1650 – MIDTERM 1 – STUDY GUIDE

This is the study guide for the exam, and is just meant to be a *guide* to help you study, just so we're on the same place in terms of expectations. Think of it more as a course summary rather than "this is exactly the questions I'm going to ask you on the exam."

1. Probability Basics (Lectures 1-2)

- Ignore the sections "Introduction" and "Classical vs. Empirical Approach"
- Write down the sample space of an experiment
- Write down an event in set notation
- Define: union, intersection, and complement
- Know the distributive laws and De Morgan's laws (you don't need to prove them)
- State the probability axioms
- Give an example of three sets A, B, C such that $A \cap B \cap C = \emptyset$ but A, B, C are not pairwise disjoint
- Show that for the unif probability distribution, we have $p = \frac{1}{n}$ (see notes)

- The antenna problem will be covered below

2. Probability and Counting (Lectures 3-4)

- Use the mn rule to count the total number of possibilities. Use this when you're counting two independent things and the order matters, like $(1,6) \neq (6,1)$
- Find the number of permutations of r objects from n objects. Use this when the order matters, like rearranging letters `PIE` or license plate numbers
- Find the number of combinations of r objects from n objects. Use this when the order doesn't matter, like `123 = 321`. There are many problems that used this method:
  - Raffle tickets
  - Undergraduate/Graduate Committee
  - Poker
  - Antenna
- Use the box and star diagram to count the different possibilities of, say, donuts in a store. You use this when the order doesn't matter and when repetition is allowed (like 4 blueberry donuts)
- State the binomial theorem. I didn't cover this in lecture but it's still important to know
- Solve a problem using multinomial coefficients, like putting 15 students in 3 different groups of 5. You don't **have** to use multinomial coefficients in those problems, it's totally ok to use

binomial coefficients instead, but in that case make sure to simplify your answer to look something like $\frac{15!}{5!5!5!}$ In my opinion it's easier to do it directly, like the way I did it in lecture.

- Finally, remember the `MISSISSIPPI` problem

3. Conditional Probability, Independence, and Bayes (Lectures 5-7)

- Define: $P(A|B)$. It's useful in practice to know both versions, the one with "Probability of A given B" and the $\frac{P(A\cap B)}{P(B)}$ version.
- Review the Bridge and Apple Pie problems from lecture
- Show that two events are independent. Sometimes the $P(A|B) = P(A)$ version is easier, and sometimes the $P(A\cap B) = P(A)P(B)$ version is easier
- You don't need to define the multiplicative law, but know how to use it
- Define: Additive Law
- Use a tree diagram to solve a problem. This is useful when you have several cases and the events are not independent
- State Bayes' Theorem, but you don't need to prove it. This is useful if you want to calculate $P(A|B)$ but you know $P(B|A)$
- Know the law of total probability and the general Bayes' Theorem. Those are useful when you can partition your universe into disjoint events/classes, like "Ball in Urn 1" and "Ball in Urn 2"

- There are many problems that use this method, like
  - ▶ Urn Problem
  - ▶ Card Game Problem
  - ▶ HIV Case Study, I would provide you $P(H) = 0.00376$

4. Random Variables (Lectures 8–9)

- Define: Random Variable, $(Y = y)$, probability mass function $p(y)$, sample space induced by a random variable
- Find $p(y)$, either as a table or as a histogram
- Find the expected value of a discrete random variable
- Use the method of indicators to find expected values, like the concierge problem in lecture or the marble problem on the homework. You usually use this method when you have n people and when it is hard to calculate $P(X = i)$ directly.
- Find $E(g(X))$ where g is a real valued function
- Find the variance of a discrete random variable
- Know the magic variance formula and know how to derive it
- Beware that in general we don't have $\text{Var}(X + Y) = \text{Var}(X) + \text{Var}(Y)$ but it is true if the random variables are independent

5. Distributions (Lectures 10–13)

- Define: Bernoulli trial, Bernoulli Random Variable.
- You use a Bernoulli random variable is if you have **one** experiment, like **one** coin toss with two outcomes, success and failure
- Find the expectation and var of a Bernoulli Random Variable
- Define: Binomial Distribution and know how to find $p(y)$
- You use a binomial distribution if you have a **sequence** of Bernoulli trials, like a sequence of coin flips, and you count the number of successes.
- Find the expectation, variance of a Binomial Random Variable and show that the sum of the probabilities is 1
- Review the pea flower problem from lecture
- Define: Geometric Distribution and know how to find $p(y)$
- You use the geometric distribution if you want to know how long it takes to get your first success, like how long it takes to flip heads for the first time
- Know the formula of the geometric series
- Find the expectation of a Geometric Random Variable and show that the sum of probabilities is 1. You don't need to prove the formula for the variance
- Review the computer crashing problem
- Know the memoriless property of the geometric distribution, that is, the geometric distribution "forgets" about the first m trials, as if we started from scratch

- Define: Hypergeometric distribution and know how to find $p(y)$
- Know how to do the marble problem in the Hypergeometric distribution lecture
- You use the hypergeometric distribution to count the number of successes when there is no replacement. Compare this with the binomial distribution where there is replacement
- Define: Poisson Distribution but you don't need to know how to find $p(y)$
- You don't need to know how to construct the Poisson distribution
- Find the expectation of a Poisson Random Variable and show that the sum of probabilities is 1. You don't need to prove the formula for the variance
- You use the Poisson distribution to count the number of events that occur during a fixed time interval. Check out the customers example in lecture.