APMA 1650 – MIDTERM 1

1. *(5 points)* A witness in a hit-and-run accident tells the police that the license plate of the car in the accident starts with the letters RE and contains digits 4 and 5, like `REX745` or `REE544`. How many different license plates match this description? Write your answer as the product of two numbers. Here a license plate contains 3 letters followed by 3 digits

Answer:

☐ Work on Scratch Paper

2. *(5 points)* The oracle of Delphi is known to speak the truth 2 out of 3 times. It throws a die and reports that the number obtained is a six. Find the probability that the number obtained is actually a six. Please simplify your answer.

Answer:

☐ Work on Scratch Paper

3. *(5 points)* In how many ways can a class of 15 students be divided into 5 groups of 3 students in such a way that the two students named "Ben" are not in the same group? Assume the order of the groups doesn't matter. Write your answer in terms of factorials, but you don't need to simplify your answer.

Answer:

□ Work on Scratch Paper

4. *(5 points)* An ice cream parlor serves 10 flavors of ice cream. Alice and Bob each randomly and independently choose 3 out of those 10 flavors. Use the method of indicators to find the expected number of flavors chosen by both Alice and Bob. Simplify your answer.

   Note: Each person has to choose 3 different flavors (Chocolate-Chocolate-Vanilla is not ok) but their flavors are allowed to overlap (Chocolate-Vanilla-Strawberry for Alice and Chocolate-Blueberry-Mint for Bob is ok). The order of flavors doesn't matter.

Answer:

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5. *(5 points)* Suppose that, on average, 2 out of 7 people are left-handed. Assume that each person is either left-handed or right-handed, but not both. You do not need to simplify your answer. **For each part, specify which distribution you used.** On the next page, you can find a table of distributions

   (a) What is the probability that exactly 49 out of the 100 people are left-handed?

   (b) What is the probability that at least 3 people are left-handed?

   (c) Suppose the 100 people are standing in line, and you ask each of them whether they are left or right-handed. What is the probability that the 40th person you asked will be the first one who says they're left-handed?

   (d) What is the expected number of people you have to ask until you find your first left-handed person?

(a)

(b)

(c)

(d)

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Discrete Distributions

Distribution	Parameters	Probability Mass Function (pmf)	Mean	Variance
Binomial	n, p	$p(y) = \binom{n}{y} p^y (1-p)^{n-y}$ $y = 0, 1, \ldots, n$	np	$np(1-p)$
Geometric	p	$p(y) = (1-p)^{y-1} p$ $y = 1, 2, \ldots$	$\dfrac{1}{p}$	$\dfrac{1-p}{p^2}$
Poisson	λ	$p(y) = \dfrac{e^{-\lambda} \lambda^y}{y!}$ $y = 0, 1, 2, \ldots$	λ	λ

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