

APMA 1650 – MORE PRACTICE QUESTIONS

Note: Those problems taken from past exams from when I taught Discrete Math back in 2016-2017

Problem 1: Suppose there are two urns. Urn 1 contains two black balls (labeled B_1 and B_2) and one white ball. Urn 2 contains one black ball and two white balls (labeled W_1 and W_2). Suppose the following experiment is performed: One of the two urns is chosen at random. Next a ball is randomly chosen from the urn. Then a second ball is chosen at random from the same urn without replacing the first ball.

- (a) What is the probability that two black balls are chosen?
- (b) What is the probability that two balls of opposite color are chosen?

Problem 2: A telephone number is formed using 7 digits (from 0 to 9). What is the probability that a randomly chosen seven-digit phone number would have at least one repeated digit?

Problem 3: (there will be no T/F questions on the exam) Label the following statements as **TRUE (T)** or **FALSE (F)**

- (a) There are $\frac{9!}{4}$ different ways to arrange the letters in the word TABRIZIAN.
- (b) If you toss a fair coin 8 times, the probability of getting ≥ 4 heads is $\frac{1}{2}$.

- (c) Suppose you toss a fair coin and you win \$3 if you get H and you win \$1 if you get T , and suppose it costs \$2 to play this game, then the expected value of your game is \$0.

Problem 4: What is the probability that

- (a) The top and bottom cards of a randomly shuffled deck of 52 cards are both aces?
- (b) A five-card poker hand contains the ace of hearts?
- (c) A five-card poker hand contains a full house (= 3 cards of the same **denomination** and 2 cards of the same **denomination**).

Problem 5: Suppose that one out of every 1000 Pokémon is shiny, and a Pokéball with the following properties:

- If the Pokémon is shiny, the chances of catching it is 0.1 percent.
- If the Pokémon is not shiny, the chances of catching it is 50 percent.

Prof. Oak tells you that there is a magic potion that doubles the HP of your Pokémon, but the only way to get it is by winning the following game: Suppose you toss a crooked coin with the probability of having H is the probability of a Pokémon being shiny given that you catch it. You win if, out of 10 tosses, you get exactly 3 heads. What is the probability of getting the magic potion?

Problem 6: Suppose Dunkin' Donuts offers 6 different kinds of donuts: Chocolate, Cinnamon, Powdered Sugar, Boston Cream, Jelly, and Apple Cider. Today they only have 10 Chocolate donuts left but 40 each of the other kinds of donuts.

- (a) How many different selections of 20 donuts are there?
- (b) Suppose in addition to only having 10 Chocolate donuts, it only has 8 Powdered Sugar Donuts. How many different selections of 20 donuts are there?

Problem 7: What is the expected value of the number of aces in a 5-card poker hand?

Problem 8: Suppose you have a bag of 12 crooked coins (6 red and 6 blue). The red ones come up heads $\frac{3}{5}$ of the time and the blue ones come up tails $\frac{2}{3}$ of the time. Suppose you reach into the bag, pull out a coin at random, and toss it. The coin comes up tails. What is the probability that you pulled out a blue coin?

Problem 9: Oh noes!!! The Dark Lord Bun-ondorf stole Peyam's two fluffy bunnies Oreo and Cookie, and escaped to his evil fortress. You try to enter it, but unfortunately the main door is locked. Next to you, you find a well with the following inscription: "Hey, listen! If you throw in 15 diamonds in this well, then a fairy will appear and will open the door for you. The order you throw in the diamonds doesn't matter, but you need to throw in at least 2 red diamonds and at least 1 blue diamond" Next to you, you also find a bag with 20 Red diamonds, 20 Blue diamonds, 20 Yellow diamonds, 5 Green diamonds, and 5 Purple diamonds. How many different ways can you throw the diamonds in the well?

Problem 10: Suppose your poker hand contains at least two aces. What is the probability that it contains all four aces?

Problem 11: Urn 1 contains 10 red balls and 25 green balls, and Urn 2 contains 25 red balls and 15 green balls. A ball is chosen as follows:

First an urn is selected by tossing a crooked coin with probability of 0.4 of landing heads. If the coin lands heads, the Urn 1 is chosen; otherwise Urn 2 is chosen. Then a ball is picked at random from the chosen urn. If the chosen ball is green, what is the probability that it was picked from Urn 1?

Problem 12: Today, Café Peyam serves Apple, Blueberry, Chocolate, and Yam Pies. How many different selections of 20 pies contain at least 8 Apple Pies and at most 5 Blueberry Pies?

Problem 13:

(a) Show that

$$\sum_{k=0}^n k \binom{n}{k} \left(\frac{1}{2}\right)^{n-1} = n$$

Hint: First use the binomial theorem to find an expansion of $(x + \frac{1}{2})^n$. Then differentiate both sides with respect to x , and finally let $x = \frac{1}{2}$.

(b) Use your answer in (a) to find the expected number of heads when n (fair) coins are tossed.

Problem 14: When you are not studying for class, you work part-time as a barista at a local coffee shop. Customers arrive at your coffee shop at an average rate of 10 customers per hour.

(a) Model this problem with an appropriate probability distribution. What is the probability that fewer than 5 customers arrive in a fixed, one-hour period?

- (b) What is the probability that 5 or more customers arrive in a fixed, one-hour period?

Suppose it takes 4 minutes to serve one customer. The total service time is the number of minutes during a fixed, one-hour period which are spent serving customers.

- (c) What is the average total service time?
- (d) What is the variance of the total service time?