APMA 1650 – HOMEWORK 7

Problem 1: Use the definition of expected value to show that if Y_1 and Y_2 are two independent continuous random variables with joint density function $f(y_1, y_2)$ then

$$E(Y_1 Y_2) = E(Y_1)E(Y_2)$$

Note: Do not use Covariance for this!

Problem 2: Suppose X and Y are discrete random variables with joint probability mass function given by:

$$p(x,y) = \begin{cases} \frac{1}{3} & \text{if } (x,y) = (-1,0), (0,1), (1,0) \\ 0 & \text{otherwise} \end{cases}$$

(a) What is the covariance of X and Y?

(b) Are X and Y independent?

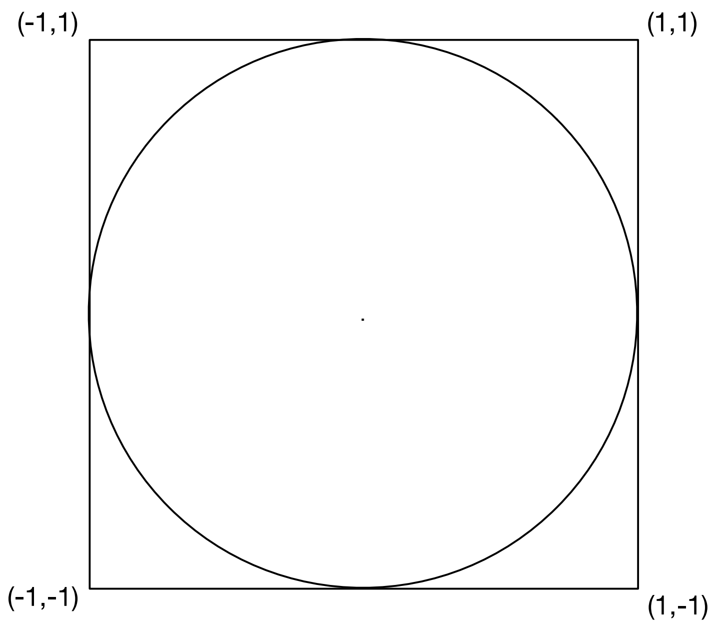
Problem 3: Take the unit circle (circle of radius 1) and inscribe it in a square (see the diagram on the next page)

You throw n darts uniformly at random at the square. Let Y be the number of darts which land in the circle. Define the "estimator"

$$\hat{\theta} = \frac{4Y}{n}$$

(a) What is the expected value and variance of $\hat{\theta}$? **(TURN PAGE)**

(b) In your own words, briefly explain how you can use this dart-throwing experiment to find an approximate value of π. Your answer should involve "Suppose you repeat that experiment many times..."



Problem 4: What do you know, another ACME widget factory question! ☺

The new line of MiniWidgets you have launched they have become so popular that your factory now has 10 machines devoted to producing MiniWidgets. The MiniWidgets produced by your machine have a mass which is normally distributed with a mean of 9 grams and a standard deviation of 0.2 grams.

(a) On a routine inspection, you take a sample of 9 MiniWidgets produced by a single MiniWidget machine. You find that the average mass of the MiniWidgets in your sample is 8.85 grams.

Do you think the MiniWidget machine is defective? (Hint: compute the probability that the sample mean is less than or equal to 8.85 grams).

(b) On another inspection, you find a machine which appears to be defective. You want to know the mean mass of widgets produced by that machine (population mean μ). To determine this, you take a sample of widgets from the machine and compute the sample mean $\bar{Y}$. Assume this machine produces widgets that are normally distributed, and that the standard deviation is the same as the other machines. How many widgets do you need to sample to be 95% confident that the sample mean is within 0.05 grams of the true population mean?

Problem 5: Let X be a binomial random variable with parameters n and p. Consider the following estimator for p:

$$\hat{p}_1 = \frac{X+1}{n+2}$$

(a) Find the bias of $\hat{p}_1$ which is defined as $E(\hat{p}_1) - p$

(b) Find the mean square error (MSE) of $\hat{p}_1$ which is bias squared plus the variance of $\hat{p}_1$