APMA 1650 – HOMEWORK 9

Problem 1: The reading on a voltmeter is uniformly distributed over the interval $[\theta, \theta + 1]$, where θ is the true voltage of the circuit. Using your voltmeter, you take n consecutive voltage readings $Y_1, \ldots, Y_n$ from a single circuit.

(a) Show that the sample mean $\bar{Y}$ is a biased estimator for θ, and compute the bias of $\bar{Y}$.

(b) Find a function of $\bar{Y}$ which is an unbiased estimator of θ.

(c) Find the MSE of $\bar{Y}$ (the biased estimator) when $\bar{Y}$ is used as an estimator of θ.

Problem 2: Let $X \sim$ Binom (n, p)

Consider the following estimator for p:

$$\hat{p}_1 = \frac{X+1}{n+2}$$

On a previous homework, we showed

$$\text{Bias}(\hat{p}_1) = \frac{1-2p}{n+2} \text{ and } \text{MSE}(\hat{p}_1) = \frac{(1-2p)^2 + np(1-p)}{(n+2)^2}$$

The standard, unbiased estimator for p is

$$\hat{p} = \frac{X}{n}$$

In class, we showed that $\text{MSE}(\hat{p}) = \frac{p(1-p)}{n}$

Find a value of p for which $\text{MSE}(\hat{p}_1) < \text{MSE}(\hat{p})$. This shows that a biased estimator can sometimes have lower MSE than an unbiased one.

Hint: Think "extreme" cases, like $p = 0$ or $p = 1$ or $p = \frac{1}{2}$

Problem 3: Let $Y_1, Y_2, \ldots, Y_n$ be a random sample from a population with probability density function parameterized by θ given by

$$f_\theta(y) = \begin{cases} \theta y^{\theta-1} & 0 < y < 1 \\ 0 & \text{otherwise} \end{cases}$$

where $\theta > 0$ is the parameter of interest.

(a) Show that the sample mean $\bar{Y}$ is an unbiased estimator for $\frac{\theta}{\theta+1}$

(b) Show that the sample mean $\bar{Y}$ is a consistent estimator for $\frac{\theta}{\theta+1}$

Problem 4: Let $Y_1, Y_2, \ldots, Y_n$ be a random sample from a population with probability density function parameterized by θ given by

$$f_\theta(y) = \begin{cases} (\theta+1) y^{\theta} & 0 < y < 1 \\ 0 & \text{otherwise} \end{cases}$$

where $\theta > -1$ is the parameter of interest

(a) Find an estimator for θ using the method of moments.

(b) Find the the maximum likelihood estimator (MLE) for θ. Compare this with your answer from (a)