APMA 1650 – MIDTERM 2 – STUDY GUIDE

This is the study guide for the exam, and is just meant to be a *guide* to help you study, just so we're on the same place in terms of expectations. Think of it more as a course summary rather than "this is exactly the questions I'm going to ask on the exam."

1. Poisson Distribution (Lecture 13)

- Define: Poisson Distribution
- Skip the construction of the Poisson distribution
- Find the expected value (but not the variance) of a Poisson Random Variable and show that the sum of probabilities is 1
- You use Poisson to count the number of events that occur during a fixed time interval, like the customers example

2. Continuous Random Variables (Lectures 13-14)

Given a function $f(y)$:

- Find the value of c for which $f(y)$ is a pdf
- Find $P(a \leq Y \leq b)$
- Find the cdf $F(y)$ and use F to calculate $P(a \leq Y \leq b)$. The normal distribution is an excellent example

- Find the first/third quartiles and median of a random variable
- Find $E(Y)$ and $\text{Var}(Y)$

3. Continuous Distributions (Lectures 14-15 + 17)

- Solve problems using the uniform distribution on $[a, b]$
- Find the $E(Y)$ and $\text{Var}(Y)$ where $Y \sim$ Unif (a, b)
- Use the $Z-$table to find $P(Z \leq a), P(Z \geq a), P(a \leq Z \leq b)$
- Sometimes we write $Y \sim N(\mu, \sigma)$ or $Y \sim N(\mu, \sigma^2)$. In both cases it is implied that the variance is σ^2
- I will either provide you a $Z-$table or give you a "simplified" $Z-$table to make the calculations easier, like I might tell you that $P(Z < 1) = 0.75$ even though those may not be the actual $Z-$values
- Know the $68 - 95 - 99.7$ rule. Beware if you see those numbers, it's likely that you use that rule in this case.
- The transformation $Z = \frac{Y-\mu}{\sigma}$ is **super** important, it allows you to go from $Y \sim N(\mu, \sigma^2)$ to $Z \sim N(0, 1)$ and vice-versa
- Know the pdf of the exponential distribution. It measures the amount of time between two subsequent events
- Show that the exponential distribution integrates to 1 and prove the formulas for $E(Y)$, $\text{Var}(Y)$, $F(y)$ and the median
- Prove the memoriless property of the exponential distribution

4. Markov and Chebyshev's Inequality (Lectures 17-18)

- State and prove Markov's and Chebyshev's inequalities
- You use Markov if you don't know *anything* about Y except for its mean μ
- You use Chebyshev if you don't know *anything* about Y except for its mean μ and variance σ^2
- If you know the distribution of Y (like normal or exponential) use that distribution instead! Why use an approximate answer if you know the exact one ☺
- Prove that fact about "deviating at least k standard deviations"

5. Multivariate Distributions – discrete (Lectures 18-19)

- Find the joint distribution function $p(y_1, y_2)$ of Y_1 and Y_2
- Show that $p(y_1, y_2)$ is a valid joint distribution function
- Find the marginal distributions $p_1(y_1)$ and $p_2(y_2)$
- Find the expected values $E(Y_1)$ and $E(Y_2)$
- Find the conditional distribution $p(y_1|y_2)$
- Show that Y_1 and Y_2 are independent

6. Multivariate Distributions – continuous (Lec 19-22)

Given a function $f(y_1, y_2)$

- Find the value of c for which f is a valid joint density function
- Calculate double integrals. A picture is absolutely crucial here.
- Find $P((Y_1, Y_2) \in A)$ where A is a region in the plane
- Find the marginal densities $f_1(y_1)$ and $f_2(y_2)$
- Find the expected values $E(Y_1)$ and $E(Y_2)$
- Find the conditional density $f(y_1|y_2)$
- Find the conditional expected value $E[Y_1|Y_2 = y_2]$
- Show Y_1 and Y_2 are independent
- Know the density of the uniform distribution
- Find $E[g(Y_1, Y_2)]$ both in the discrete case and the continuous case
- Show that if Y_1 and Y_2 are independent then $E(Y_1Y_2) = E(Y_1)E(Y_2)$
- Find $\text{Cov}(Y_1, Y_2)$ and ρ
- Prove the Magic Covariance Formula
- Show that if Y_1 and Y_2 are independent, then $\text{Cov}(Y_1, Y_2) = 0$
- Know the general formula for $\text{Var}(Y_1 + Y_2)$ but you don't need to prove it

7. Sampling Distributions (Lectures 23-26)

- Skip the introduction but know the definition of $\bar{Y}$ and S^2
- Find $E(\bar{Y})$ and $\text{Var}(\bar{Y})$
- In general the distribution of $\bar{Y}$ is unknown except in the case where the $Y_i \sim N(\mu, \sigma^2)$
- Know the transformation $Z = \frac{\overline{Y}-\mu}{\frac{\sigma}{\sqrt{n}}}$. This only works if the Y_i are normal! Carefully study the ball bearing example that follows
- Know the definition of the chi-square distribution and how to use the chi-square table
- The chi square distribution is useful because **IF** the Y_i are normal because then $\left(\frac{n-1}{\sigma^2}\right) S^2$ has a chi-square distribution
- Know the method of splitting the difference
- Know the definition of the Student's $t-$ distribution and how to use the $t-$table
- The $t-$distribution is useful **IF** all the Y_i are normal and we don't know σ. If you know σ you would use the normal distribution instead!
- Know the Central Limit Theorem. This is useful if you don't know the distribution of Y_i and the sample size is large

8. Estimators (Lectures 26-28)

- Define: Estimator, Point Estimator, Interval Estimator
- Show that $\hat{\theta}$ is biased or unbiased. Two examples are $\overline{Y}$ and $\hat{p} = \frac{Y}{n}$
- Find MSE($\hat{\theta}$) and prove the Magic MSE formula. Two examples are $\overline{Y}$ and $\hat{p}$
- Check out the two examples about the difference in populations. You don't need to memorize the table
- Understand why we use $n - 1$ in the sample variance S^2. You don't need to memorize the full proof
- Construct a confidence interval, in the case of large sample size. This boils down to the Central Limit Theorem + $Z-$table