APMA 1650 – MIDTERM 2

Name	
Brown ID	
Signature	

1. (5 = 2+3 points) Let Y be a continuous random var with density

$$f(y) = \begin{cases} c(2-y) & 0 \le y \le 2 \\ 0 & \text{otherwise} \end{cases}$$

   (a) Find the value of c that makes this a valid density function.

   (b) In that case, find $E(Y)$. Simplify your answer

(a)	
(b)	

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2. ($5 = 2+3$ points) Suppose the scores for a midterm exam have mean 50. Consider a randomly selected student's score. Simplify your answer.

   (a) Find a lower bound on the prob that the score is below 65.

   (b) What is the largest standard deviation allowed to ensure that the probability that the score is between 40 and 60 is greater than or equal to 0.64 ?

(a)	
(b)	

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3. (5 = 2+3 points) Let $\hat{\theta}_1$ and $\hat{\theta}_2$ be two (independent) estimators for a parameter θ

Suppose $E[\hat{\theta}_1] = E[\hat{\theta}_2] = \theta$ and $\text{Var}(\hat{\theta}_1) = \sigma_1^2$ and $\text{Var}(\hat{\theta}_2) = \sigma_2^2$

$$\text{Let } \hat{\theta}_3 = a\hat{\theta}_1 + (1-a)\hat{\theta}_2 \qquad \text{(where } 0 < a < 1\text{)}$$

(a) Show $\hat{\theta}_3$ in an unbiased estimator of θ

(b) Find $\text{MSE}(\hat{\theta}_3)$

(a)	
(b)	

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4. (5 points) Suppose you're taking $n = 100$ iid samples $Y_1, Y_2, \cdots, Y_n$ Suppose $\overline{Y} = 50$ and $S^2 = 9$

Find a 60% confidence interval for $\overline{Y}$ as an estimator of μ. Simplify your answer. Some relevant (simplified) z values are

F(-2.3) $= 0.099$	F(-0.8) $= 0.2005$	F(-0.2) $= 0.4013$
F(0.2) $= 0.5987$	F(0.8) $= 0.7995$	F(2.3) $= 0.9901$

Answer:	

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5. (5 = 3 + 2 points) Let X and Y be random variables with joint density (assume $x \geq 0$)

$$f(x,y) = \begin{cases} cx & \text{for } 0 \leq x^2 \leq y \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

(a) Find the value of c for which $f(x,y)$ is a valid density

(b) In that case, find the marginal density $f_X(x)$

(a)	
(b)	

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(Scratch Paper)