APMA 0350 – HOMEWORK 1

Problem 1: (4 = 2 + 2 points)

(a) For which values of r is $y = e^{rt}$ a solution of

$$y'' + 3y' - 10y = 0\,?$$

(b) Determine whether $y = t^4$ is a solution of

$$t^3\, y'' + t\, y' - 8y = 0$$

Problem 2: (5 points, 1 point each) Find the order of each equation and state if it is linear or non-linear. If it's linear, state if it is homogeneous or inhomogeneous. No justification needed

(a) $e^t\, y'' + y' - t^2\, y = 13t^3$

(b) $(y')^2 + t\, y + \cos(y) = 0$

(c) $y''' + 10y'' + 5y' - 2y = 0$

(d) $\cos(t)\, y' + t\, y = 0$

(e) $y' = t^3 y^2 + t\, y$

(Turn page)

Problem 3: (3 points) Solve

$$\begin{cases} y' = 2y + 3 \\ y(0) = 3 \end{cases}$$

To do this, please divide both sides by $2y+3$ and recognize the left-hand-side as a derivative, just like we did in lecture with ln.

Problem 4: (4 = 2 + 2 points) (Mini-Theory) Consider the ODE

$$y'' - 4y' + 4y = 0$$

(a) Show that $y = Ae^{2t} + Bte^{2t}$ solves the ODE, where A and B are constants

(b) Show that $y = Ae^{2t} + Bte^{2t}$ is the only solution to the ODE

Hint: Consider $\left(e^{-2t}y\right)''$

Problem 5: (4 points) (Application) You're studying the growth of a bacteria population and you're noticing that the rate of change of the population is equal to three times the number of bacteria present. Set up an ODE for the the number of bacteria and calculate the amount of time it takes for the population to double. Does the answer you found depend on the initial population y_0 of the bacteria?

Direction fields will be part of the first programming assignment.