APMA 0350 – FINAL EXAM

Name	
Brown ID	
Signature	

1. *(5 points)* Solve and write your answer in implicit form. Don't forget to check for exactness

$$\begin{cases} \dfrac{dy}{dx} = -\left(\dfrac{3x^2y + 2y^3}{x^3 + 6xy^2}\right) \\ y(1) = 2 \end{cases}$$

Answer	

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2. *(5 points)* Solve and write your answer in explicit form

$$\begin{cases} t^4\,(y') + t^3\,y = t^4\sin(t) & t > 0 \\ y(\pi) = 2 \end{cases}$$

$y =$	

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3. *(5 points)* Solve using the method with $Dy = y'$

$$y'' - 3y' - 4y = 5e^{4t}$$

Hint: Here it's easier to use $(D^2 - 3D - 4) = (D+1)(D-4)$

Note: If you don't remember that method, you can solve it using undetermined coefficients for a max of 4 points.

$y =$	

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4. *(5 points)* Find the eigenvalues and eigenfunctions of

$$\begin{cases} y'' = \lambda y \\ y'(0) = 0 \\ y(6) = 0 \end{cases}$$

Eigenvalues	
Eigenfunctions	

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5. *(5 points)* Use Laplace transforms to solve

$$\begin{cases} y'' + 3y' + 2y = 12e^{2t} \\ y(0) = 4 \\ y'(0) = 2 \end{cases}$$

$y =$	

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6. *(5 points)* Use Laplace transforms to solve

$$\phi'(t) + 5\int_0^t e^{2(t-\tau)}\phi(\tau)d\tau = 0 \quad \text{with } \phi(0) = 6$$

$\phi(t) =$	

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7. *(5 points)* Solve $\mathbf{x}' = A\mathbf{x}$ **AND** draw a phase portrait where

$$A = \begin{bmatrix} -2 & 1 \\ -1 & -2 \end{bmatrix}$$

$\mathbf{x}(t) =$	
Portrait	

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8. *(5 points)* Use **Variation of Parameters** to solve $\mathbf{x}' = A\mathbf{x} + \mathbf{f}$

$$\text{Where } A = \begin{bmatrix} -1 & 2 \\ -1 & 2 \end{bmatrix} \text{ and } \mathbf{f} = \begin{bmatrix} e^{t} \\ 2e^{2t} \end{bmatrix}$$

$\mathbf{x}(t) =$	

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9. *(5 points)* Find and classify the equilibrium point(s) of

$$\begin{cases} x' = x - y^2 \\ y' = y - x^2 \end{cases}$$

Equilibrium Point	Classification

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10. *(5 = 2 + 3 points)* You are the CEO of PeyUSPS, a mail delivery company that has distributors in the cities Peyamgeles and Tabriziville. Packages can be sent from one city to the other, or stay at the same location. Suppose 30% of the packages in Peyamgeles are sent to Tabriziville per day, and 60% of packages in Tabriziville are sent to Peyamgeles. Let $x(t)$ and $y(t)$ be the number of packages in Peyamgeles and Tabriziville respectively, where t is in days. Assume no other packages go in/out of the two cities.

    (a) Set up an ODE model of the form $\mathbf{x}' = A\mathbf{x}$

    (b) Find the equilibrium points of (a) and their stability. Do not solve the system

(a) $A = \begin{bmatrix} & \\ & \end{bmatrix}$

(b)

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