APMA 0350 – HOMEWORK 2

Problem 1: (3 points, 1 point each) Do the following ODE satisfy the assumptions of the Existence-Uniqueness Theorem from lecture? Why or why not?

(a)

$$\begin{cases} \dfrac{dy}{dt} = \dfrac{y}{t^2+1} \\ y(-1) = 9 \end{cases}$$

(b)

$$\begin{cases} \dfrac{dy}{dt} = y^2\left(|t| + y\right) \\ y(0) = 2 \end{cases}$$

(c)

$$\begin{cases} \dfrac{dy}{dt} = \dfrac{1}{y+1} \\ y(1) = -1 \end{cases}$$

Problem 2: (4 points, 1 point each) Consider the equation

$$y' = (y+2)(y-1)(y-3)$$

(a) Find the equilibrium solutions

(b) Draw a bifurcation diagram (that table with signs)

(c) Draw a sketch of the equilibrium solutions and of a couple of solutions between them (that graph with the curves)

(d) Classify the equilibrium solutions as stable/unstable/bistable.

Problem 3: (6 points, 2 points each) Solve using separation of variables.

(a) (Implicit form)

$$\frac{dy}{dt} = \frac{t\sec(y)}{y\left(e^{t^2}\right)}$$

(b) (Remember the initial condition)

$$\begin{cases} \dfrac{dy}{dt} = \dfrac{2t + \sec^2(t)}{2y} \\ y(0) = -5 \end{cases}$$

(c) (Beware of hidden solutions, see last example in the notes)

$$\frac{dy}{dt} = \frac{-t\,(y-1)^2}{3}$$

Problem 4: (5 = 2 + 1 + 2 points, Application)

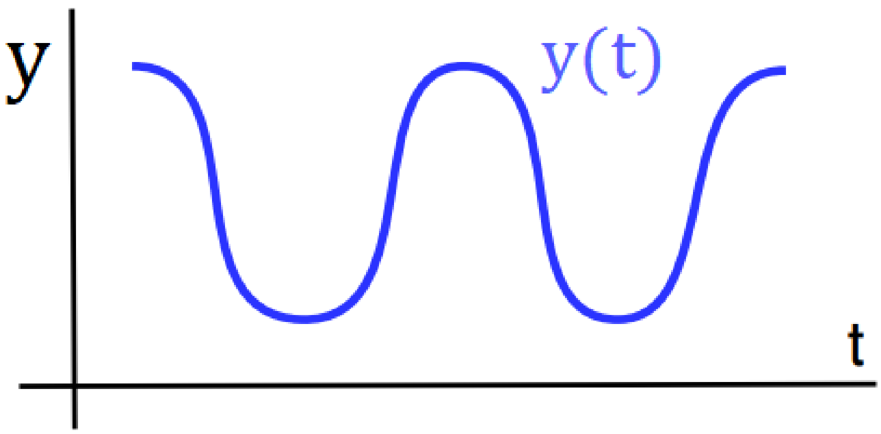
(a) Suppose you're trying to model a population of bunnies. Initially there are 100 rabbits, and you notice that the growth rate is 0.08 bunnies/day. Moreover, due to limiting resources, it seems that the bunny population doesn't exceed 1000 rabbits.

Set up a differential equations model for the number of rabbits and follow the steps from lecture to solve it. You can skip the partial fractions part if you wish.

(b) What is the limit of the solution in (a) as $t \to \infty$?

(c) For which t do we have $y(t) = 900$? Please find an exact formula for t, as well as an approximate value (using your calculator)

Problem 5: (2 points, Mini Theory) Show that the function $y(t)$ whose graph is below *cannot* be a solution of an ODE of the form $y'(t) = f(y(t))$



Hint: Find two different values t_1 and t_2 on the graph with $y(t_1) = y(t_2)$ but $y'(t_1) < 0$ and $y'(t_2) > 0$. Since the ODE is true for all t, it has to be true for t_1 an t_2 as well. Use that to find a contradiction