APMA 0350 – HOMEWORK 3

Problem 1: (4 points, 2 points each) Solve the following ODE using integrating factors

(a)
$$y' = 2y + t^2 e^{2t}$$

(b)
$$ty' + 2y = \sin(t)$$

Problem 2: (5 = 1 + 2 + 2 points)

For this problem, refer to the Chemical Tanks example from lecture

Suppose a tank is filled initially (at time $t = 0$) with 100 gallons of fresh clean water.

Water containing 10 grams/gallon of chemical pollutants enters the tank at a rate of 2 gallons/day, and the mixture in the well-stirred tank leaves the tank at the rate of 1 gallon/day.

(a) Find the amount of water $W(t)$ as a function of time and determine the time when that amount reaches 200 gallons. In order to do this, either set up a *very easy* ODE for $W(t)$ or use your intuition about the problem.

(b) Write down the differential equation that describes the total amount of pollutants $P(t)$ in the tank and find the initial condition $P(0)$ of this quantity at time $t = 0$. Again, think in terms of rate in minus rate out. **(TURN PAGE)**

(c) Find the total amount of pollutants in the tank at the time you determined in (a)

Problem 3: (7 = 1 + 1 + 2 + 2 + 1 points)

For this problem, refer to the Rabbits vs Foxes example in the lecture notes, as well as the Savings example.

Our goal in this problem is to develop a model for the bunny population at Brown by taking the following factors into account:

- The initial population on Jan 1 was 50 rabbits
- On average, every rabbit has 0.1 offspring per month
- On average, 30% of the existing rabbit population dies per month
- 30 rabbits migrate into the Brown campus area per month.

Using this information:

(a) Clearly identify your dependent and independent variables, including their units

(b) Derive a differential equation for the rabbit population that takes all factors listed above into account, using the trick with h small and calculating the change, similar to the Savings example in lecture.

(c) Perform a qualitative analysis: Find the equilibrium solutions, draw the bifurcation diagram, plot the equilibrium solution and at least two other solutions, and determine if the equilibrium solution is stable/unstable/bistable **(TURN PAGE)**

(d) Solve the ODE, including the initial condition

(e) Use your solution to figure out what happens to the population of bunnies as $t \to \infty$.

Problem 4: (4 points)

For this problem, you need to use **Newton's law of cooling** which states that the rate of change of the temperature of an object is proportional to the difference between that temperature and the ambient temperature. Recall that a is proportional to b if there is k such that $a = kb$.

Peyam baked a Yam Pie, and he only wants to eat it once the temperature reaches 25 degrees Celsius, so it will not burn him. Initially he measured the temperature and the pie was 40 degrees Celsius. One minute later, the pie was 35 degrees. The ambient temperature of the room is 22 degrees. When should Peyam start eating the pie?

Note: It's ok to use your calculator and approximate values here!