APMA 0350 – HOMEWORK 5

Problem 1: (2 points) Find the general solution of an ODE whose auxiliary equation is

$$5r^2(r+4)^3(r+7)\left(r^2+9\right)^3\left(r^2+2r+10\right)^2=0$$

Problem 2: (5 points) Find the eigenvalues and eigenfunctions of

$$\begin{cases} y'' = \lambda y \\ y'(0) = 0 \\ y(3) = 0 \end{cases}$$

Note: I recommend reviewing the "Mixed Example" in the lecture notes, although this is slightly different. Remember that $\omega > 0$. That should help you figure out if you start with $m = 0$ or $m = 1$.

Problem 3: (4 points) Use undetermined coefficients to solve

$$\begin{cases} y'' - 5y' + 4y = 20\cos(2t) + 30\sin(2t) \\ y(0) = 1 \\ y'(0) = 3 \end{cases}$$

Problem 4: (2 points, 0.5 point each)

Guess the form of the particular solution (see the section "Who's that Particular Solution?" in the lecture notes)

(a) $y'' - 3y' + 2y = e^t$

(b) $y'' - 3y' + 2y = t^2e^{2t}$ **(TURN PAGE)**

(c) $y'' - 2y' + 5y = \sin(2t)$

(d) $y'' - 2y' + 5y = e^t \cos(2t)$

Problem 5: (4 = 2 + 2 points, Mini Theory) Use the differential operators method (the one with $Dy = y'$) to solve the following ODE

(a) $y'' - 4y' + 4y = e^t$

(b) $y'' - 5y' + 6y = e^{3t}$

Notice how there is no guesswork involved here, which is what makes this method so nice ☺

Note: Here is a video solving part (b)

Video: Cool Inhomogeneous Equations

Problem 6: (3 points, Application)

Suppose an object of mass $m = 2$ is attached to a spring with constant $k = 1$. Moreover assume there is damping $\gamma = 2$ due to friction, and no forcing term. Assuming the initial displacement is 5 and the initial velocity is 3, solve for the displacement $y(t)$ and describe in your own words what happens to the motion of the spring.