APMA 0350 - Programming Assignment Solutions

February 26, 2024

1. Use the dfield app to draw the direction field of

$$y' = y(y+3)$$

On that direction field, please click on three solutions, one in the region y > 0, one in the region -3 < y < 0, and one in the region y < -3.

dy/dt	= y*(y+3)									sin((x), n	ot 2)	and	sin	x).	110 w 2	s a00	utui	gonon	leure, logarithille alle e
The D	Displa	ay:																			
Minimum t: -10 Minimum y: -5 Ai							Arro	arrow length:			15	5 Variable length arrows					1				
Maxi	mun	ıt:	10		Max	imun	1 y:	5		Nun	iber (of an	rows	20							
Grap	h Dire	ction	Field																		
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35	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	
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-3	1	1	1	1	1	-	1	1	1	1	1	1	1	1	1	1	1	1	1	1	
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2. Use Python to apply Euler's Method with N = 50 on [2, 3] where

 $\begin{cases} y' = \cos(y) + ty \\ y(2) = 5. \end{cases}$

No need to print the (t, y) values but please plot the points on a graph.

```
Solution:
               import numpy as np
               from matplotlib import pyplot as plt
               #Initial condition values
               t0 = 2
               y0 = 5
               #Final value of interval you are solving over
               tf = 3
               #Stepsize = deltat
               n = 50
               deltat = (tf - t0)/n
               #Time and y variable arrays
               t = np.linspace(t0,tf,n)
               y = np.zeros([n])
               #Initial condition
               y[0] = y0
               #Method calculation
               for i in range(1,n):
                   y[i] = deltat*(np.cos(y[i-1]) + (y[i-1]*t[i-1])) + y[i-1]
               #Printing results - not necessary for your homework
               for i in range(n):
                   print(t[i], y[i])
               #Plotting
               plt.plot(t,y,'o')
               plt.xlabel('Value of t')
               plt.ylabel('Value of y')
               plt.title("APMA 0350 - Forward Euler's Method")
               plt.show()
```



3. Use the dsolve command in Python to solve the following. Don't solve them by hand.

(a)

$$y' + y = 3\cos(2t)$$

Solution: #Problem 3 from sympy import * t = symbols('t') y = Function('y') deq = diff(y(t),t) + y(t) - 3*cos(2*t) ysoln = dsolve(deq,y(t)) print(ysoln) Eq(y(t), C1*exp(-t) + 6*sin(2*t)/5 + 3*cos(2*t)/5)

(b)

$$\begin{cases} y' + 2y = 2te^{2t} \\ y(0) = 1 \end{cases}$$

Solution:

Problem 3 - Using dsolve on function in b)

```
#PROBLEM 2#
t=symbols('t')
y=Function('y')
deq=diff(y(t),t)+2*y(t)-2*t*exp(2*t)
ysoln=dsolve(deq,y(t),ics={y(0):1})
print('Solution is',ysoln)
```

Solution is Eq(y(t), ((4*t - 1)*exp(4*t)/8 + 9/8)*exp(-2*t))

(c)

$$\begin{cases} y' = 20y(1 - \frac{y}{20}) \\ y(0) = 10. \end{cases}$$

Please also plot the solution in (c), using -5 and 5 as the t limits and -1 and 21 as the y limits.

Solution:

Problem 3 - Using dsolve on function in c)

t=symbols('t')
y=Function('y')
deq=diff(y(t),t) - 20*y(t)*(1-y(t)/20)
ysoln=dsolve(deq,y(t),ics={y(0):10})
plot(ysoln.rhs,(t,-5,5),ylim=[-1,21])

