APMA 1650 – FINAL EXAM

Name	
Brown ID	
Signature	

1. *(6 points)* What is the expected number of occurrences of the string `PEYAM` in a random string of length 10? For example, in `ABRAPEYAMC` the number of occurrences is 1 and for `PEYAMPEYAM` it is 2. We're using letters from the 26-letter English alphabet.

Answer:	

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2. *(6 points)* The company PeyAmazon has 50 employees. Each month, it sends out a "Happy birthday" email to each employee whose birthday is in that month (if there's a month with no birthdays, no email is sent that month). Suppose each employee's birthday is equally likely to be in any of the 12 months of the year, independently of the birthdays the others. In a year, what is the expected number of months in which the company sends out > 1 birthday email? Use the method of indicators.

Answer:	

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3. *(6 points)* In a bunny sanctuary, the number of baby bunnies born per month Y follows a distribution (distri-bun-tion?) with $E(Y) = 3$ and $\text{Var}\ (Y) = 2$. Each baby bunny needs to be provided with a cozy nest, incurring a cost of \$2 per bunny, along with a fixed overhead cost of \$10 for the necessary supplies. Let C represent the total cost of providing nests for the new baby bunnies in a randomly selected month. Find an interval within which C is expected to lie with a probability of at least 0.92

   Note: The confidence interval techniques won't work here because we don't know the sample size

Answer:	

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4. *(6 points, 2 points each)* Suppose X and Y are random variables with a joint uniform distribution inside the triangle with vertices

$$(0,0) \quad (1,0) \quad (0,1)$$

(a) Find the joint pdf $f(x, y)$ of X and Y

(b) Find Cov (XY)

(c) Are X and Y independent?

(a)	
(b)	
(c)	

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5. *(6 points, 2 points each)*

Let $X_1, \cdots, X_n$ and $Y_1, \cdots, Y_m$ be iid Poi (λ) and let

$$\hat{\lambda} = a\left(\frac{X_1 + \cdots + X_n}{n}\right) + b\left(\frac{Y_1 + \cdots + Y_m}{m}\right) \text{ where } a, b > 0$$

(a) Find an equation for a and b so that $\hat{\lambda}$ is unbiased for λ

(b) In that case, calculate MSE $(\hat{\lambda})$

(c) In that case, with $m = n$, show $\hat{\lambda}$ is consistent for λ

(a)	
(b)	
(c)	

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6. *(6 points)* Suppose you want to measure the mean tastiness μ of donuts, where μ is in units of Pe-yums. In order to do so, you taste $n = 16$ donuts and find a sample mean of 9 Pe-yums. Assume the tastiness of the samples is normally distributed. What is the largest sample standard deviation you can measure in order for the length of a 90% confidence interval to be less than 4?

Note: Here is a table of relevant (simplified) $t-$values

df \ p	0.25	0.2	0.15	0.1	0.05	0.025	0.02	0.01
15	0.6	0.8	1	1.5	2	2.1	2.2	2.6
16	0.7	0.9	1	1.3	1.7	2	2.2	2.6
17	0.7	0.9	1	1.3	1.8	2	2.2	2.6

Answer:	

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7. *(6 points)* Suppose $Y_1, Y_2, \cdots, Y_n$ are iid $\sim$ Binom (m, p)

Find the Maximum Likelihood Estimator of p. Simplify your answer and write it in terms of $\overline{Y}$

Note: Here n is the sample size and m is the number of trials

Answer:	

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8. *(8 points, 2 points each)* Suppose the ages of people in the continent of Peyamerica are normally distributed with variance 25. You sample 100 people and find an average age of 28. You claim that the average is < 30

   (a) Set up the 4 elements of a hypothesis test

   (b) Is your claim supported at the $\alpha = 0.05$ level? Here you can use $P(Z \leq -2) = 0.05$

   (c) What is the $p-$value of the test? Write your answer in the form $P(Z \leq c)$

   (d) If our Alt hypothesis is that the average is 26, what is β? Write your answer in the form $P(Z \geq c)$

(a)	
(b)	
(c)	
(d)	

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