APMA 1941G – HOMEWORK 5

Problem 1: (10 = 8 + 2 points)

This problem refers to Example 1 "Rapidly Oscillating Coefficients"

(a) In lecture, we defined

$$\overline{a} = \int_0^1 a + 2w'a + wa'dy$$

where $w = w(y)$ is the solution of (here $' = \frac{d}{dy}$)

$$\begin{cases} -(aw')' = a' \\ w(0) = w(1) \end{cases} \qquad (\star)$$

Show that

$$\overline{a} = \left(\int_0^1 \frac{1}{a(y)} dy \right)^{-1}$$

Hint:

- First solve the equation $(\star)$ for w by antidifferentiating (be careful about the limits of integration). You may also assume that[1] $w'(0) = 0$

[1] This is not a very restrictive assumption actually, because by the assumption $w(0) = w(1)$ and Rolle's Theorem, there is a point x_0 in $(0,1)$ such that $w'(x_0) = 0$, and you can easily adapt the proof you've given with this condition instead of $w'(0) = 0$

- Then go back to the definition of $\overline{a}$, integrate by parts to write wa' in terms of $w'a$ (why are the boundary-terms 0?), and use use your formula for w.

(b) [This should be quick] Using your formula for $\overline{a}$ from (a), find the general solution of

$$-\overline{a}u^0_{xx} = f(x)$$

(don't worry about the periodicity for this part).

Note: The reason I put this part is to show you that in fact, one can find an explicit formula for our function u_0.

Problem 2: (10 points, 2.5 points each) This is pure ODE, but it's one of our steps in Example 2 "An Oscillator with Damping"

(a) Using undetermined coefficients, find a particular solution of

$$w''(t) + w(t) = -\frac{1}{4}\cos(3t)$$

(b) Using undetermined coefficients, find a particular solution of

$$v''(t) + v(t) = -\frac{3}{4}\cos(t)$$

(c) Use $-\cos^3(t) = -\frac{3}{4}\cos(t) - \frac{1}{4}\cos(3t)$ to find the general sol of

$$u_1''(t) + u_1(t) = -\cos^3(t)$$

(d) Find the solution of (c) that satisfies $u_1(0) = 0$ and $u_1'(0) = 0$