APMA 0350 – HOMEWORK 6

Problem 1: (4 points, 2 points each) Use variation of parameters to find the general solution of the following ODE. Simplify your answers.

(a) $t^2 y'' + 4t^2 y' + 4t^2 y = e^{-2t}$

(b) $t^2 y'' - t(t+2) y' + (t+2)y = 2t^3$

Note: For (b) assume t and te^t are solutions of the hom. equation

Problem 2: (4 points, 2 points each)

Find the general solution of $y'' + y = \cos(t)$

(a) Using undetermined coefficients

(b) Using variation of parameters

Make sure your answers match

Problem 3: (2 points, Mini Theory)

In this problem, we'll rederive the Var of Par equations, but for

$$y'' - 5y' + 6y = f(t)$$

Where $f(t)$ is an inhomogeneous term

Recall that the hom. solution is $y_0 = Ae^{2t} + Be^{3t}$ **(TURN PAGE)**

Variation of Parameters: Suppose y_p is of the form

$$y_p = u(t)e^{2t} + v(t)e^{3t}$$

And suppose for simplicity that

$$e^{2t}u'(t) + e^{3t}v'(t) = 0$$

Calculate $(y_p)'$ and $(y_p)''$ and plug into the ODE to show

$$\left(2e^{2t}\right)u'(t) + \left(3e^{3t}\right)v'(t) = f(t)$$

Problem 4: (2 points) Use tabular integration to find $\mathcal{L}\left\{t^2\right\}$

Problem 5: (2 points) Use complex exponentials to find

$$\mathcal{L}\left\{\cos(3t)\right\} \text{ and } \mathcal{L}\left\{\sin(3t)\right\}$$

Problem 6: (2 points) Find examples of functions $f(t)$ and $g(t)$ with

$$\mathcal{L}\left\{f(t)g(t)\right\} \neq \mathcal{L}\left\{f(t)\right\}\mathcal{L}\left\{g(t)\right\}$$

Most guesses should work, but try out constant/exponential functions

Problem 7: (4 points) Use Laplace Transforms to solve

$$\begin{cases} y'' + 9y = \cos(2t) \\ y(0) = 1 \\ y'(0) = 0 \end{cases}$$