APMA 0350 – MIDTERM 2 – STUDY GUIDE

This is the study guide for the exam, and is just meant to be a *guide* to help you study, just so we're on the same place in terms of expectations. Think of it more as a course summary rather than "this is exactly the questions I'm going to ask you on the exam."

There will be **NO** coding on the exam, but there might be proofs, like on the homework

Format: There are 4 questions on the exam, all of them free response, no multiple choice. Two questions will be about second-order ODE and two questions will be about Laplace transforms.

A Laplace transform table will be provided, see course website. No need to print it out. You're also allowed one 2-sided 8.5×11 cheat sheet

Useful trig identities to know:

(1) $\sin^2(x) + \cos^2(x) = 1$

(2) $1 + \tan^2(x) = \sec^2(x)$

(3) $\cos(-x) = \cos(x), \sin(-x) = -\sin(x)$

(4) $\cos(2x) = \cos^2(x) - \sin^2(x)$, $\sin(2x) = 2\sin(x)\cos(x)$

(5) $\cos^2(x) = \frac{1}{2} + \frac{1}{2}\cos(2x)$, $\sin^2(x) = \frac{1}{2} - \frac{1}{2}\cos(2x)$

Useful integration techniques to know:

(1) Integrals of $\sin, \cos, \tan, \sec$

(2) $\int \frac{1}{x^2+1} dx = \tan^{-1}(x) + C$

(3) $u-$substitution

(4) Integration by parts, but I'll only ask you about easy cases like $\int xe^x dx$

(5) $\int \cos^2(x)dx$, $\int \sin^2(x)dx$; check out this video in case you forgot

(6) You need to know partial fractions to do Laplace transforms

Second-Order ODE (Lectures 11 – 18)

- Solve $ay'' + by' + cy = 0$ using the method with $Dy = y'$ There are also some nice variations of this on the homework, with repeated roots and inhomogeneous equations
- Solve $ay'' + by' + cy = 0$ using auxiliary equations
- I could ask you about higher order differential equations, but in that case would give you the roots beforehand
- Find the eigenvalues and eigenfunctions of the boundary-value problem $y'' = \lambda y$ The lecture notes and the homework problem give you more or less all possible cases. Don't forget to check what the starting value of m is.
- Use undetermined coeffs to find a particular solution to a ODE:

▶ **Rule 1:** If the right hand side is e^{rt} guess Ae^{rt}

- ▶ **Rule 2:** If the right hand side is a polynomial like t^2 you guess $At^2 + Bt + C$ (the most general version)
- ▶ **Rule 3:** cos goes with sin and vice-versa

- If the root of the right-hand-side coincides with the root of y_0, then there is resonance and you add an extra t
- The section on mechanical vibrations is just practice with the techniques. You don't need to know the physical background and you don't need to know the formulas for amplitude
- Solve an ODE using variation of parameters and **make sure that the coefficient of y'' is 1**
- Be able to rederive the var of par formula. I've done an example of that in lecture and there is also one on your homework.

Laplace Transform (Lectures 19 – 25)

- Find $\mathcal{L}\{f(t)\}$ using the definition of Laplace transform.
- I recommend the method of Tabular integration, it allows you to calculate $\mathcal{L}\{t^4\}$ very quickly. Here is an example of this method.
- Prove the formulas for $\mathcal{L}\{f'(t)\}$ and $\mathcal{L}\{f''(t)\}$
- Solve a second-order ODE using Laplace transforms. I could ask you zero or nonzero initial conditions. Notice that the auxiliary equation should appear somewhere in your calculation
- Write a given function in terms of step functions u_c. The jumps could be constants, or entire functions.

- Find the Laplace transform of u_c, and know the formula

$$\mathcal{L}\left\{f(t-c)u_c(t)\right\} = e^{-cs}\mathcal{L}\left\{f(t)\right\}$$

- This also works in reverse, for example $e^{-5s}\mathcal{L}\left\{t^3\right\} = \mathcal{L}\left\{(t-5)^3\, u_5(t)\right\}$

- Find the laplace transform of $e^{2t}\sin(3t)$ where you "shift" the Laplace transform.

- Also know how to do it in reverse, like find a function whose Laplace transform is $\frac{1}{(s-2)^2+1}$

- This sometimes requires to complete the square, like $s^2+4s+5 = (s+2)^2+1$

- You may have to combine both, like the $\frac{3(s-2)e^{-3s}}{s^2-4s+5}$ example from lecture.

- Solve ODEs with jumps. This means first write $f(t)$ in terms of jump functions (if not already done) and take Laplace transforms

- Feel free to write your solutions in terms of $h(t)$, like in lecture, but you **need** to define what $h(t)$ is!

- Solve ODE involving Dirac Delta

- The definition of convolution will be provided but know that $\mathcal{L}\left\{f \star g\right\} = \mathcal{L}\left\{f\right\}\mathcal{L}\left\{g\right\}$

- Find the Laplace transform of a function that is written as a convolution, like the Laplace transform of $\int_0^t (t-\tau)^2 e^{\tau}\, d\tau$

- Find an inverse Laplace transform and express your answer as an integral, like the inverse Laplace transform of $\left(\frac{1}{s^2+1}\right)\left(\frac{1}{s^2+4}\right)$. I would explicitly tell you "Leave your solution as an integral"
- Use Laplace transforms and convolution to solve an ODE in terms of integrals. An excellent example is $y'' - 4y' + 4y = \tan(t)$ I would explicitly tell you "Leave your solution as an integral"
- Solve integral equations and integro-differential equations (see lecture)
- Solve the integral question with

$$\int_0^1 x^3(1-x)^7 dx$$