APMA 0350 – HOMEWORK 8

Problem 1: (2 points) Write the ODE in the form $\mathbf{x}' = A\mathbf{x} + \mathbf{f}$

$$y''' + 2y'' - 4y' + y = 5t$$

Do **NOT** solve it!

Problem 2: ($6 = 3 + 3$ points) Solve the following systems. Do **NOT** draw a phase portrait

(a)

$$\mathbf{x}' = A\mathbf{x} \text{ where } A = \begin{bmatrix} 1 & -2 \\ 3 & -4 \end{bmatrix}$$

(b)

$$\mathbf{x}' = A\mathbf{x} \text{ where } A = \begin{bmatrix} 5 & -1 \\ 3 & 1 \end{bmatrix} \text{ and } \mathbf{x}(0) = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$$

Problem 3: (4 points) Solve the system **and** draw a phase portrait by hand

$$\mathbf{x}' = A\mathbf{x} \text{ where } A = \begin{bmatrix} 3 & -2 \\ 2 & -2 \end{bmatrix}$$

Problem 4: (4 points) Solve $y'' - 5y' + 6y = 0$ by writing it as a system $\mathbf{x}' = A\mathbf{x}$ and solving that system. Do **NOT** use another method to solve this.

Hint: First solve for $\mathbf{x}$ and then use the fact that $y = x_1(t)$

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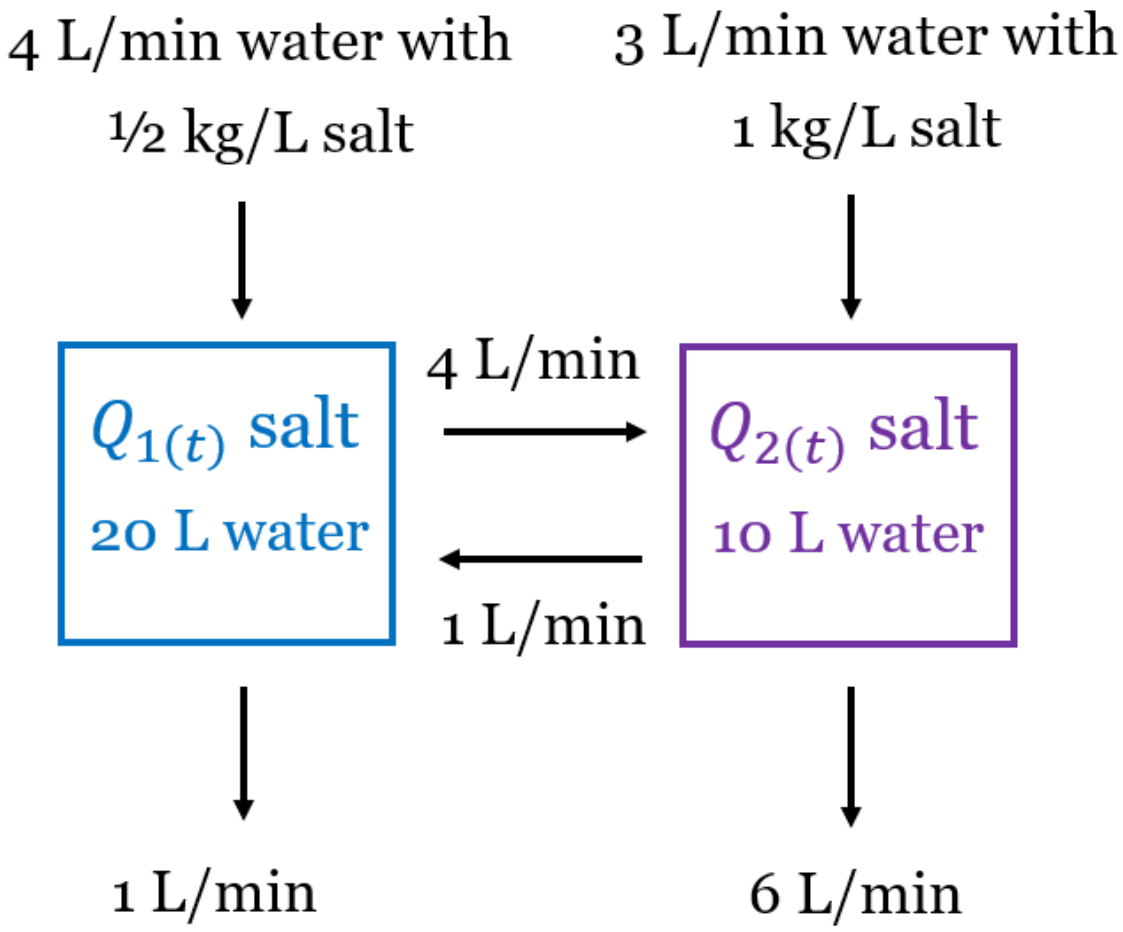
Problem 5: (4 points) Consider the following system of interconnected tanks that have an inflow and outflow of salt-water mixture.



Set up but do **NOT** solve a system of ODE of the form

$$\mathbf{Q}'(t) = A\mathbf{Q}(t) + \mathbf{b}$$

Where $\mathbf{Q}(t) = \begin{bmatrix} Q_1(t) \\ Q_2(t) \end{bmatrix}$ with $Q_1(t)$ the amount of salt in tank 1 and $Q_2(t)$ the amount of salt in tank 2 and $\mathbf{b}$ is a constant vector **(TURN**

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Note: For simplicity, assume that the amount of water in each tank is constant.

Hint: For each tank, carefully think about how much salt goes in/out and whether that amount depends on Q_1 or Q_2 or not. It might help to think in terms of units, you want kg/min everywhere.