APMA 0350 – HOMEWORK 9

Problem 1: (8 points, 4 points each) Solve the following systems and draw a phase portrait

(a)
$$\mathbf{x}' = A\mathbf{x} \qquad A = \begin{bmatrix} -1 & -4 \\ 1 & -1 \end{bmatrix}$$

(b)
$$\mathbf{x}' = A\mathbf{x} \qquad A = \begin{bmatrix} 1 & -1 \\ 1 & 3 \end{bmatrix}$$

Problem 2: (8 points, 4 points each) Solve the following systems. Do **NOT** draw a phase portrait

(a)
$$\mathbf{x}' = A\mathbf{x} \qquad A = \begin{bmatrix} -3 & 2 \\ -1 & -1 \end{bmatrix} \qquad \mathbf{x}(0) = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$$

(b)
$$\mathbf{x}' = A\mathbf{x} \qquad A = \begin{bmatrix} 3 & -2 \\ 8 & -5 \end{bmatrix} \qquad \mathbf{x}(0) = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$$

Note: Write your solution in (b) in the form $e^{\lambda t} \begin{bmatrix} at + b \\ ct + d \end{bmatrix}$ where a, b, c, d are integers

Problem 3: (4 points) Use variation of parameters to find the general solution of $\mathbf{x}' = A\mathbf{x} + \mathbf{f}$ where
$$A = \begin{bmatrix} 2 & -1 \\ 3 & -2 \end{bmatrix} \qquad \mathbf{f} = e^t \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$