

## APMA 1941G – HOMEWORK 9

### Problem 1: (10 points)

Consider the following ODE on  $(0, 1)$ , where  $u^\epsilon = u^\epsilon(x)$

$$\begin{cases} \epsilon u_{xx}^\epsilon + u_x^\epsilon = 2x \\ u^\epsilon(0) = 1, u^\epsilon(1) = 1 \end{cases}$$

We expect there to be a boundary layer at  $x = 0$

Follow the method used in lecture to find a good approximation  $u^*$  of  $u^\epsilon$  that incorporates the boundary layer

**Note:** You only need to limit yourself to the  $O(1)$ -terms. For the matching-part, you may use any method that you wish.

### Problem 2: (10 points)

Consider the following ODE on  $(0, 1)$ , where  $u^\epsilon = u^\epsilon(x)$

$$\begin{cases} \epsilon u_{xx}^\epsilon + u_x^\epsilon + u^\epsilon = 0 \\ u^\epsilon(0) = 0, u^\epsilon(1) = 1 \end{cases}$$

We expect there to be a boundary layer at  $x = 0$

Follow the method used in lecture to find a good approximation  $u^*$  of  $u^\epsilon$  that incorporates the boundary layer (**TURN PAGE**)

**Note:** This time, go up to the  $O(\epsilon)$ -terms. For the matching part, you may use any method you wish (but I think you'll be forced to apply Method 2).