

SMUGGLING AND THE BLACK MARKET FOR FOREIGN EXCHANGE

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This paper develops a model of the black market for foreign exchange in which black market currency is demanded and supplied by firms engaged in smuggling. It is demonstrated that quotas or foreign exchange rationing are not necessary to induce the coexistence of legal and black foreign exchange markets. Variants of the model analyze the implications of capital flight, unbalanced trade, quotas, foreign exchange licensing, and economies with more than two goods.

1. Introduction

Beginning with the pioneering work of Bhagwati and Hansen (1973), a number of investigators have analyzed illegal transactions in commodity trade using the standard two-good trade theoretic model. One phenomenon often present in countries with illegal commodity trade is a black market for foreign exchange. The relationship between smuggling and this black market has been discussed by Bhagwati (1964, 1967), and Sheikh (1976), but the link between smuggling and the behavior of this black market has not been formally set out with respect to an explicit theory of smuggling. Perhaps this reflects the difficulty of introducing currencies and exchange rate phenomena into the common framework of smuggling's analysis, the two-good, two-country, single-period barter model of trade.

In this paper the black market for foreign exchange is modeled as reflecting the demand for foreign exchange to purchase illegal import and the supply of foreign exchange derived from illegal export. The model developed differs markedly from that of Sheikh. He applied to the case of foreign exchange control the partial equilibrium analysis of Boulding (1947), Bronfenbrenner (1947), and Michael (1954), for a single consumption commodity in a domestic market subject to price control and rationing. In our model, legal foreign exchange is treated as an input into a linearly homogeneous illegal trading technology common to competing firms. As a result, the analysis of its rationing is inherently different from that of consumption goods. In the general equilibrium model of trade presented here, the determination of the black market exchange rate does not depend

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on assumptions concerning resale and the form of rationing as in the analysis of Sheikh. Furthermore, the view that quotas or foreign exchange rationing are necessary for the creation of a black market for foreign exchange is found to be false.

The formal model of the black market for foreign exchange is developed in section 2 by replacing the relative prices of the standard two-by-two barter model of trade with nominal money prices denominated in two currencies. It is demonstrated that in a model of smuggling where legal trade cloaks smuggling activity [Pitt (1981)], the black market for foreign exchange will coexist with a legal exchange market even in the absence of quotas or rationing. In section 3, the assumptions of the standard two-by-two barter model are relaxed to allow for capital flight, unbalanced trade, foreign exchange restrictions, and quotas. In section 4, the results are extended to the case of more than two goods. Section 5 briefly summarizes the results.

2. Conditions for the coexistence of legal and black markets for foreign exchange

2.1. Smuggling and black markets in the standard two-by-two barter model

Our initial objective is to determine necessary and sufficient conditions for the coexistence of legal and black markets for foreign exchange and whether these conditions differ between alternative models of smuggling. In particular, are tariffs and quotas, which are sufficient to induce smuggling, also sufficient to induce black market foreign exchange transactions? The analysis uses the framework of the smuggling literature, the standard two-by-two barter model of trade. This model is then 'denormalized' to admit money prices and two currencies. This involves replacing the unique free trade terms of trade for each good expressed in terms of the other with nominal prices for each good expressed in terms of units of foreign exchange. The relative price of the small country, which may reflect a tariff or quota, is likewise replaced by two nominal prices expressed in terms of units of domestic exchange. Obviously, the usual relative prices are simply the ratios of these nominal prices. If it is required that exporters surrender the declared value of exports and importers receive legal foreign exchange equal to the declared value of import, then legal trade flows and legal foreign exchange sales are equal. As black market foreign exchange will exist only to finance illegal trade, the coexistence of legal and black markets for foreign exchange requires that legal trade and smuggling coexist.¹ This simple result is very basic in understanding the

¹Note that if we allowed for foreign exchange demands in addition to that generated by the need to finance commodity trade, such as capital flight, then restrictions on satisfying those demands will provide incentives for black markets independently of any tariffs or quotas levied on commodity trade. Below, we will specifically allow for these types of demands and examine their implications. Here, we wish to determine the minimum set of assumptions under which black markets will exist, and hence, foreign exchange is assumed to be used for commodity trade only.

relationship between smuggling models and the black market for foreign exchange.

In their pioneering paper, Bhagwati and Hansen (1973), modeled smuggling as trade at world prices, in the sense that trade taxes are avoided, but involving a less favorable rate of transformation than free trade as a result of the real resource costs incurred in smuggling. In their base case of constant costs, one would expect to observe smuggling or legal trade, but not both. If the tariff exceeds the cost of smuggling, only smuggling will be observed; and if these costs exceed the tariff, only legal trade will be observed. Thus, except for the special circumstance where the tariff exactly equals the cost of smuggling, the Bhagwati-Hansen model cannot generate coexisting legal and black market foreign exchange transactions solely as a result of a tariff.

A quota is not equivalent to a tariff in this regard. Falvey (1978) has extended the Bhagwati-Hansen model for the case of a quota and demonstrated that legal and illegal trade may coexist. Foreign exchange will be traded legally to finance trade under the quota and illegally to finance any additional trade. Perhaps it is this conceptualization that underlies assertions that a tariff in the absence of quotas or foreign exchange rationing does not create incentives for a black market and that such a market can only exist if there are such controls.

A model of smuggling in which a tariff alone is sufficient to result in the coexistence of smuggling and legal trade may imply that quotas or rationing are not necessary for the coexistence of black market and legal foreign exchange transactions. Such a model of smuggling has been formalized in Pitt (1981), with a very general framework which recognizes that some legal trade may be necessary for smuggling to occur. Obviously, in order to underinvoice an import, it is necessary to declare some positive value. Furthermore, it is logical to assume that success in avoiding detection is inversely related to the degree of underinvoicing. For example, a \$100 item declared to be worth \$99 has a greater chance of fooling the customs officer than the same item declared at \$10. This reasoning extends to other types of faked declarations as well. A trader seeking to undervalue a good whose unit price is well known may do so by declaring it to be something else or may declare an incorrect number of units. For example, 10 000 tons of rubber may be declared to weigh 9000 tons. Such an illegal declaration has a greater probability of success than one of 1000 tons. Alternatively, grade I rubber may be declared as grade II, or with greater risk of detection, as grade V. In all the above cases the declared value of a shipment is legal trade, while smuggling is the difference between the actual and the declared values.² The

² Michael (1954), in his classic analysis of black market behavior, also recognized the role of legal activity in hiding illegal activity. He commented that 'increase in production for the black market...will often be accompanied by some increase in the amount offered legally, since it is difficult to hide the additional product completely from the authorities; some of it must be offered legally' (footnote, p. 631).

type of invoice faking is not of consequence here, although it can be important in the detection of smuggling.³ The point is that the risk of detection and thus the cost of smuggling is greater, the less is the ratio of the declared value to the actual value.

2.2. The barter model of smuggling

Consider a small economy which produces two goods in perfect competition. They are an exportable x and an importable m . Identical firms trade illegally according to the barter smuggling function

$$\bar{s}_x = g(l_x, s_x), \quad (1)$$

where

- \bar{s}_x is the quantity of good x successfully smuggled,⁴
- l_x is the quantity of good x legally traded,
- s_x is the quantity of good x input into the smuggling activity,
- $g(\cdot)$ is a concave, twice continuously differentiable linear homogeneous function. Furthermore, it has the properties:

$$g_l \geq 0, \quad (2)$$

$$1 \geq g_s \geq 0, \quad (3)$$

$$s_x > \bar{s}_x. \quad (4)$$

Eq. (2) states that the marginal product of legal trade in the smuggling activity is non-negative. Eq. (3) states that a unit increase in smuggling input results in a positive but less than unit increase in successful smuggling. Legal trade and smuggling coexist because legal trade is an input to smuggling, and therefore also a cost to smuggling. The (non-negative) private cost of smuggling is the difference between ex ante smuggling, s_x , and ex post smuggling, \bar{s}_x . The equilibrium price and equilibrium production resulting from smuggling with a function such as $g(\cdot)$ does not depend on whether these costs are real resource costs to the economy, perhaps resulting from higher transportation costs in illegal trade, or are merely transfers to the government treasury resulting from fines levied against shipments in which smuggling has been detected, or are transfers to customs officials in the form of bribes.⁵

³On this point, see Bhagwati (1981).

⁴As this is a barter model, flows of legal and illegal trade could equivalently be expressed in terms of the importable m .

⁵Welfare, of course, does depend on whether the costs of smuggling are real resource costs or merely transfers. On this point, see Pitt (1981). Also note that we are ignoring the possibility of revenue-, rent-, or bribe-seeking as a resource diverting activity as in Krueger (1974), and Bhagwati and Srinivasan (1980).

An equivalent representation of this model would have firms maximizing expected profit given a function describing the probability of being caught. Note that the linear homogeneous smuggling function (1) can be written as:

$$\bar{s}_x = s\hat{g}(l_x/s_x), \quad (1')$$

where $\hat{g}(\cdot)$ is the original function $g(\cdot)$ transformed through division by s_x . The function $\hat{g}(l_x/s_x)$ is bounded by zero and one and can be thought of as the probability of smuggling success, an increasing function of the legal component of one's trade.⁶

In addition to the coexistence of legal trade and smuggling, a model of trade in which smuggling takes places with the technology (1) is characterized by *price disparity*, defined as a domestic price which is less than (exceeds) the cost of legal import (export). Price disparity means that legal trade seemingly occurs at a loss.

Again, restricting our analysis to the standard two-by-two trade model, if legal trade requires the sale or purchase of legal foreign exchange, the coexistence of legal trade and smuggling imply the coexistence of legal and black markets for foreign exchange. Importers who underinvoice in order to avoid tariffs will need to obtain illegally an amount of foreign exchange equal to the difference between the actual and the declared value of their shipments. Exporters who underinvoice to avoid export taxes will possess illegally earned foreign exchange which they will be willing to sell. The market formed by these illegal traders is referred to as the 'black market for foreign exchange'.

2.3. The formal model of the black market for foreign exchange

Equilibrium in this market can be characterized by denormalizing the two-by-two barter model of smuggling presented in Pitt (1981) (and briefly described above) so as to admit money prices in two currencies. Since legal foreign exchange purchases (sales) are assumed identically equal to the declared (legal) value of imports (exports), in the absence of quantitative restrictions the market for legal foreign exchange can be characterized as perfectly competitive with domestic exchange freely convertible into legal

⁶The model of smuggling presented above has been built on the notion that both legal and illegal trade occur through legal channels (customs houses) via misinvoicing. The model also makes sense when legal trade is through legal channels and illegal trade is through illegal channels, with trading firms handling both sources of trade. Indeed, Bhagwati (1981) is of the view that this model makes more sense when illegal trade is through illegal channels. The legal transactions make it possible for traders to reduce the risk of detection on the purchases and sales of illegally obtained goods. The smuggling function (1') is perhaps more amenable to this conceptualization. In this case, illegal trade cannot take place without legal trade, while in practice it does in countries with total prohibitions on trade. Thus, our model may not be the most general representation of illegal trade outside of legal channels.

foreign exchange at some market clearing price for foreign exchange in terms of domestic exchange, E . Similarly, as there are no quantitative restrictions on sales or purchases in the black market, it is cleared at a price B of foreign exchange in terms of domestic exchange.

Exporters maximize profit (in domestic currency units, in this case) given by:

$$\pi_x = p_x^w B g(l_x, s_x) + p_x^w E(1 - t_x) l_x - p_x^d (s_x + l_x), \quad (5)$$

where l_x and s_x represent legal and ex ante smuggled export, $g(l_x, s_x)$ is the export smuggling function discussed above, p_x^w and p_x^d are the world and domestic prices of the exportable, respectively.

Similarly, importers maximize profit given by:

$$\pi_m = p_m^d h(l_m, s_m) + p_m^d l_m - p_m^w E(1 + t_m) l_m - p_m^w B s_m, \quad (6)$$

where $h(\cdot)$ is the import smuggling function having all the same properties of $g(\cdot)$ above.

First-order conditions for the exporter with respect to l_x and s_x are:

$$p_x^w B g_l + p_x^w E(1 - t_x) = p_x^d, \quad (7)$$

$$p_x^w B g_s = p_x^d, \quad (8)$$

and first-order conditions for the importer with respect to l_m and s_m are

$$p_m^d h_l + p_m^d = p_m^w E(1 + t_m), \quad (9)$$

$$p_m^d h_s = p_m^w B. \quad (10)$$

Conditions (7)–(10) state that the cost of an additional unit of tradeable to the firm will just equal the revenue to be earned by trading it, be it legally or illegally. These conditions have been expressed in domestic currency units in (7)–(10), but could have been equivalently expressed in any other units. Also note from (7) and (9) that price disparity will exist for both commodities.

As profits will be zero, (5) and (6) can be set equal to zero and solved for the domestic prices of the exportable and importable as the weighted average of all trade. Setting world prices equal to unity and dividing (5) by l_x and (6) by l_m , these domestic price equations are:

$$p_x^d = \frac{B \hat{g}(\hat{s}_x) + E(1 - t_x)}{\hat{s}_x + 1}, \quad (11)$$

$$p_m^d = \frac{B\hat{s}_m + E(1+t_m)}{\hat{h}(\hat{s}_m) + 1}, \quad (12)$$

where $\hat{g}(\cdot)$ and $\hat{h}(\cdot)$ are the smuggling functions in terms of $\hat{s}_x = s_x/l_x$ and $\hat{s}_m = s_m/l_m$. Maximizing (11) and minimizing (12) is equivalent to maximizing profits and results in first-order conditions:

$$\hat{B} - \frac{1-t_x}{(\hat{s}_x + 1)\hat{g}' - \hat{g}} = 0, \quad (13)$$

$$\hat{B} - \frac{(1+t_m)\hat{h}'}{\hat{h} - \hat{s}_m\hat{h}' + 1} = 0, \quad (14)$$

where $\hat{B} = B/E$, $\hat{g}' = \partial\hat{g}(\hat{s}_x)/\partial\hat{s}_x$ and $\hat{h}' = \partial\hat{h}(\hat{s}_m)/\partial\hat{s}_m$.⁷

For notational convenience, eq. (13), the supply curve for black market foreign exchange given a tax rate, is written as:

$$\hat{B} = \sigma(\hat{s}_x), \quad (15)$$

and equation (14), the demand curve for black market foreign exchange given a tariff rate, is written as

$$\hat{B} = \delta(\hat{s}_m). \quad (16)$$

We can readily sign the derivatives $\sigma' = \partial\sigma(\hat{s}_x)/\partial\hat{s}_x > 0$ and $\delta' = \partial\delta(\hat{s}_m)/\partial\hat{s}_m < 0$, allowing us to draw supply and demand curves for black market foreign exchange as in fig. 1.⁸

Note that in this model the supply of black market foreign exchange is simply the value (quantity) of ex post export smuggling, $g(l_x, s_x)$, and the demand is the value (quantity) of ex ante import smuggling s_m . The equilibrium conditions in the markets for foreign exchange are

$$l_x = l_m: \text{equilibrium in the legal foreign exchange market}, \quad (17)$$

$$g(s_x, l_x) = s_m: \text{equilibrium in the black market for foreign exchange}, \quad (18)$$

⁷From this point on, references in the text to the black market exchange rate, import smuggling, and export smuggling refer to the normalized terms \hat{B} , \hat{s}_m , and \hat{s}_x .

⁸Fig. 1 has been drawn over the space of two quadrants since t_x or t_m may have negative values representing subsidies. As smuggling is defined here as declared value less actual value, overinvoicing represents 'negative' smuggling.

Note that the normalized demand curve represented in fig. 1 is negatively sloped over its entire range, unlike the black market demand curves of Michael (1954) and Sheikh (1976). As a result, the stability of equilibrium is assured.

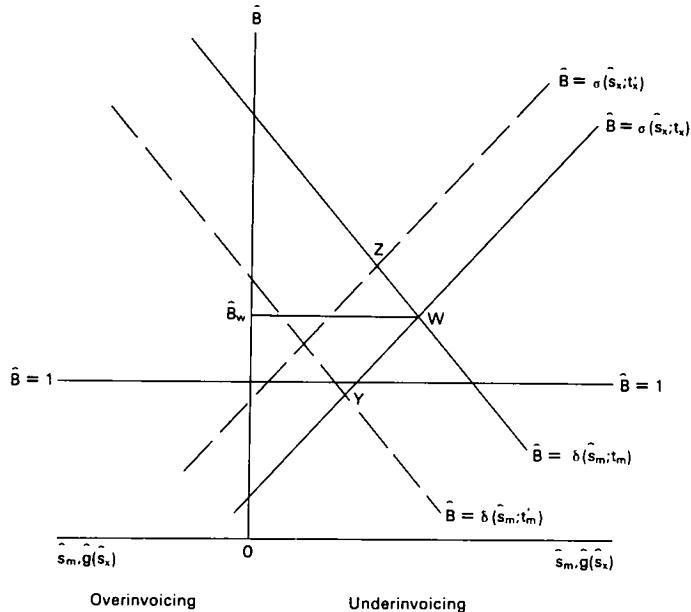


Fig. 1

which together imply a single normalized equilibrium condition

$$\hat{g} = \hat{s}_m. \quad (19)$$

Eqs. (15), (16) and (19) constitute a system of three equations in three unknowns. In fig. 1, equilibrium is at W with exchange rate \hat{B}_w where condition (19) is satisfied. Equilibrium at W illustrates the case of a positive black market premium ($\hat{B} > 1$). The black market price of foreign exchange can be less than the legal price with a lower rate of import tariff $t'_m < t_m$, which shifts the demand curve for black market foreign exchange leftward with a new equilibrium at Y .⁹ In general, reductions in the import tariff reduce import and export smuggling as well as the black market rate of exchange. In fig. 1, a reduction in the export tax from t_x to t'_x also results in a fall in the level of smuggling but increases the relative black market rate of exchange. Note that only one of the goods needs to be taxed for both to be smuggled and exhibit price disparity.¹⁰

⁹It is straightforward to prove that \hat{B} can be less than unity. Note that if $t_x = 0$ and $t_m = 0$, then the supply and demand curves illustrated by fig. 1 will intersect at 1 on the vertical \hat{B} axis. Any positive export tax will shift the supply curve $\hat{B} = \sigma(\hat{s}_x)$ rightward, so that the new equilibrium \hat{B} is less than one.

¹⁰There will not be smuggling if one good is taxed in the special case where the rate of subsidy on the other good is such that the supply and demand curves of fig. 1 intersect on the vertical axis.

How can there be concurrently active legal and black markets for foreign exchange with a price differential? If the legal exchange rate were less than the black market rate, why would any exporter trade legally and, as required by the exchange control, sell those foreign exchange earnings in the market for an apparent loss vis-à-vis their value in the black market? This question is equivalent to that posed and answered in the barter model of smuggling [Pitt (1981)]: why does legal trade take place if the domestic price of an export exceeds the return to legal export, that is, legal export takes place at a loss? The reason has already been clearly stated — legal transactions hide illegal activities from detection and thereby reduce the costs associated with them. For example, by declaring a positive value at customs for a taxed export, firms are choosing to convert part of their foreign exchange earnings at the inferior legal effective (i.e. tax inclusive) exchange rate so as to reduce the costs of smuggling and converting the undeclared and illegal value of the export at the higher and tax-free black market exchange rate.¹¹ Note, however, that these results depend critically on our particular formulation of smuggling. As we have shown, profit maximizing illegal trade with this smuggling function requires some legal transactions. As a result, quotas or foreign exchange rationing are not necessary for the existence of a black market in foreign exchange.

3. Some extensions: capital flight, unbalanced trade, quotas, and exchange control

The assumptions explicit or implicit in the standard two-by-two barter model of trade used in the analysis so far include balanced trade and no capital flows or desire to hold cash balances either domestically or abroad (capital flight). These are of course important phenomena often associated with black markets for foreign exchange. If one allows for foreign exchange demands in addition to that generated by the need to finance commodity trade, then restrictions on satisfying those demands will provide incentives for black markets independently of any tariffs or quotas levied on commodity trade. Below, our model of smuggling and black market behavior is extended to examine the effects of exogenous changes in the demand and supply of foreign exchange arising from capital flight and trade deficits.

3.1. Capital flight

The effect on commodity smuggling and the black market exchange rate of a change in the exogenous demand for black market foreign exchange,

¹¹ Michael also suggests that firms will voluntarily sell some output at the inferior legal price when the legal and black markets coexist. In noting that production for the black market is often accompanied by additional legal sales meant to hide illegal transactions, he notes that these legal sales take place at an apparent loss 'since the difference between the marginal cost of these units that are offered at the official market, and the (low) price received for them, is an additional cost of the added output produced for the black market' (footnote, p. 631).

perhaps for purposes of capital flight, can be determined by rewriting the equilibrium conditions (17) and (18) to include those exogenous demands. The new equilibrium condition in the black market including an exogenous demand F is:

$$g(l_x, s_x) = s_m + F; \quad (18a)$$

or, normalizing exogenous demand on the value of legal trade:

$$\hat{g} = \hat{s}_m + f, \quad (19a)$$

where f is exogenous demand expressed as a share of legal trade. Totally differentiating the black market supply (15), demand (16), and equilibrium conditions (19a), we have:

$$d\hat{B} - \sigma' d\hat{s}_x = 0, \quad (20)$$

$$d\hat{B} - \delta' d\hat{s}_m = 0, \quad (21)$$

$$\hat{g}' d\hat{s}_x - d\hat{s}_m - df = 0. \quad (22)$$

Recalling that $\sigma' = \partial\sigma(\hat{s}_x)/\partial\hat{s}_x > 0$, $\delta' = \partial\delta(\hat{s}_m)/\partial\hat{s}_m < 0$, and $\hat{g}' > 0$, we can sign the derivatives of interest:¹²

$$\frac{ds_m}{df} = \frac{\sigma'}{\hat{g}'\delta' - \sigma'} < 0 \quad (23)$$

$$\frac{d\hat{s}_x}{df} = \frac{\delta'}{\hat{g}'\delta' - \sigma'} > 0 \quad (24)$$

$$\frac{d\hat{B}}{df} = \frac{\sigma'\delta'}{\hat{g}'\delta' - \sigma'} > 0 \quad (25)$$

From eqs. (23), (24) and (25) it can be seen that an increase in the exogenous demand for black market foreign exchange reduces normalized import smuggling, increases normalized export smuggling, and increases the normalized black market exchange rate. The increase in exogenous black market demand will 'crowd out' some demand by import smugglers by increasing the cost of illegal import relative to legal import. The higher black

¹²A geometrical representation of the effects of exogenous changes in the demand and supply of foreign exchange is not presented because it involves the rescaling of the horizontal axis of fig. 1.

market premium on earnings will elicit an increase in the rate of export underinvoicing.¹³

3.2. Trade deficits

In a similar fashion, the balanced trade assumption of our model can be relaxed by altering the equilibrium conditions. Allowing for a deficit as a proportion r of nominal legal import yields a nominal legal exchange market equilibrium condition of

$$l_x = (1 - r)l_m, \quad (17a)$$

which implies a normalized equilibrium condition of

$$(1 - r)\hat{g} = s_m. \quad (19b)$$

To evaluate the effect of an increase in the trade deficit, the supply, demand, and equilibrium conditions are differentiated and solved. The derivatives of interest are:

$$\frac{d\hat{s}_m}{dr} = \frac{\sigma' \hat{g}}{\hat{g}' \delta'(1 - r) - \sigma'} < 0, \quad (26)$$

$$\frac{d\hat{s}_x}{dr} = \frac{\delta' \hat{g}}{\hat{g}' \delta'(1 - r) - \sigma'} > 0, \quad (27)$$

$$\frac{d\hat{B}}{dr} = \frac{\sigma' \delta' \hat{g}}{\hat{g}' \delta'(1 - r) - \sigma'} > 0. \quad (28)$$

Eqs. (26), (27), and (28) demonstrate that increasing the rate of trade deficit reduces normalized import smuggling, but increases normalized export smuggling and the black market foreign exchange premium. Note that the signs of these derivatives are the same as for an increase in exogenous foreign exchange demand (capital flight). It is perhaps somewhat surprising that augmenting the supply of legal foreign exchange increases the black market premium and the level of export smuggling. This result seemingly contradicts the popular view that black market premia exist because of excess demand in the legal market, so that increasing supplies in the legal market will reduce the premia in the black market. This view is correct for rationed

¹³The positive relationship between capital flight and the black market exchange rate is one explanation for the observed positive premium typically associated with black market foreign exchange. Even in the absence of capital flight, an import bias in the trade regime would accomplish the same result.

commodities in a closed economy [cf. Michaely (1954)], in which case increasing supplies in the legal market eventually must drive excess demands to zero. However, in an economy in which illegal trade occurs in order to avoid tariffs, the incentive to smuggle will exist no matter how much legal foreign exchange is added to supplies. An increase in the trade deficit will increase the supply of legal exchange and thereby reduce the price of legal exchange relative to black market exchange. The higher black market premium increases the relative cost of import smuggling, thereby reducing \hat{s}_m , but provides an additional incentive to generate black market foreign exchange through export smuggling, thereby increasing \hat{s}_x .

3.3. Quotas and exchange control

In the two-by-two model, a quantitative restriction on the sale of foreign exchange is equivalent to a quota since foreign exchange sales are directly proportional to the level of import and export (plus any deficit). In the studies of Michaely (1954), and Sheikh (1976), there was a great deal of concern with the form of rationing for goods whose supply was subject to quantitative restriction. In both papers the basic models developed assumed that there was no resale of the rationed good. Upon relaxing this assumption, Sheikh found that some results changed fundamentally. By not permitting resale, Michaely and Sheikh were forced to make assumptions concerning the method of rationing. The choice of rationing method is somewhat arbitrary (Michaely and Sheikh chose different methods), and yet for them, black market equilibrium depends crucially upon this choice. This is because in their models the black market is a residual market since only demands which are unsatisfied by the ration of the legal market are translated into black market demands. Thus, the shape of the black market demand curve will be different if the rationed commodity tends to be allocated to those who value it most, so that the residual demand consists primarily of those who value the commodity less, compared with a rationing scheme which allocates primarily to those with a low value of the rationed commodity, so that the residual demand consists of those with high valuations. Note that if (costless) resale were permitted, all rationing systems would result in the same black market price.

There is a fundamental difference between a model such as our's, where the rationed good, legal foreign exchange, may be thought of as an input, and Michaely's, where the rationed good is consumed. If a consumer were to receive a ration in excess of his demand at its black market price, resale would be required in order to maximize utility. Consumers attach lower values to incremental units. In our model, firms which import choose as inputs into the trading process levels of legal and illegal trade (l_m and s_m) so as to maximize profits [given by (6)]. Rationing legal foreign exchange is

equivalent to fixing l_m for a firm at its ration level. The linear homogeneity of the smuggling function means that input ratios $s_m/l_m = \hat{s}_m$ are independent of the level of the ration. This is readily seen in eq. (12) where the firms' objective function has been rewritten in terms of only one choice variable, \hat{s}_m . All values of \hat{s}_m are feasible for any fixed (positive) value of l_m , and all firms will choose the same input ratios \hat{s}_m no matter what their ration. All firms will value any and all units of the rationed foreign exchange identically, so that the presence or absence of resale and the method of rationing is irrelevant. The constant returns to scale assumption simply means that all firms, irrespective of size, face the same terms of trade in their legal/illegal trade. A firm which receives a large ration of foreign exchange does not have to resell some of it to realize the premium associated with this ration. The firm can do just as well by simply using this foreign exchange in international trading since the value of the quota right to it is as great as to any other firm. The aggregate level of legal trade, attempted and successful smuggling, and all domestic relative prices are unaffected by alternative allocations of a given total amount of legal foreign exchange among firms.

Having established that the ability to resell quota rights and the method by which they are rationed are irrelevant to the black market for foreign exchange,¹⁴ it is evident that quotas are equivalent to tariffs in our model. For every quota there exists a tariff or export tax which yields a level of legal trade equal to the quota. Higher quotas correspond to lower tariffs (or export taxes). Thus, the effects of a fall in the quota on the normalized levels of smuggling and black market exchange rate are the same as a rise in the tariff, the effects of which are illustrated in fig. 1.

4. Trading economies with more than two goods

Assume that in the two-good economy just described, a third good, an importable, comes into existence and is assigned a tariff rate by the government. Traders will smuggle this third good, call it z , only if the costs of joint smuggling/legal trade per unit of import are not greater than the costs of wholly legal trade. If the smuggling function for z is $\hat{f}(\hat{s}_z)$ having the same properties as $\hat{g}(\cdot)$ and $\hat{h}(\cdot)$, given by (2), (3), and (4), smuggling will take place only if

$$1 + t_z \geq \hat{B}/\hat{f}, \quad (29)$$

¹⁴The allocation of quotas may be relevant to the prices of *goods* in production models where quotas are used to allocate traded intermediate inputs among firms which produce a commodity output. Firms which receive quota rights for two or more traded inputs in suboptimal proportions to each other or to fixed factors will have incentives to enter the black market for goods. However, even here the equilibrium black market price of *foreign exchange* need not depend on the inter-firm allocation of the quota.

where t_z represents the tariff rate for z . The left-hand side of (29) represents the domestic price of z with wholly legal trade, and the right-hand side represents the minimum cost of import for a unit of joint legal/illegal trade (10). If $1 + t_z < \hat{B}$, that is, if the tariff is less than the black market premium, then good z will not be smuggled since the derivative \hat{j}' satisfies $0 \leq \hat{j}' \leq 1$. If t_z is sufficiently large so that z is smuggled, new equilibrium values for \hat{B} will be established.¹⁵

Following from the above, if good z is not smuggled, a reduction in its tariff has no effect on the relative black market exchange rate. It is seen in (13) and (14) that \hat{B} depends only on the smuggling functions and trade taxes of smuggled goods. Increasing the tax on the exportable may induce the smuggling of the previously nonsmuggled import z because it reduces the equilibrium black market rate of exchange. Furthermore, a decrease in the tariff of the currently smuggled import m may also induce the smuggling of previously nonsmuggled good z for the same reason. Finally, in the example above, for a sufficiently high tariff on good z , good m may cease to be smuggled if the black market exchange rate is bid higher than its smuggling break-even level.

These results for three goods are directly generalizable to any arbitrary number of goods. Not all taxed goods will be smuggled, depending on their break-even condition (29).¹⁶ An increase in the import tariff of a smuggled good may increase \hat{B} , cause some smuggled imports to become wholly legally traded and some nonsmuggled exports to be smuggled. A decrease in the export tax of a smuggled export may result in it no longer being smuggled, but may lead to the commencement of smuggling of another export. The black market exchange rate may rise and some previously nonsmuggled importables may be smuggled.

5. Summary

This paper has modeled the black market for foreign exchange as reflecting the demand for foreign exchange to purchase illegal import and the supply of foreign exchange available from illegal export. The role of the black market exchange rate in equilibrating the demand for smuggled import and export is made explicit. It was found that in a model of smuggling where legal transactions cloak illegal transactions, thereby reducing their cost, legal and black foreign exchange markets may coexist even in the absence of quotas or

¹⁵Note that the equivalence of tariffs and quotas demonstrated for the two-by-two model hold also for economies with more than two goods and more than one quota. For example, take an economy with three goods (x, m, z), two of which are importables subject to quota. As before, the value of any quota right received by a firm is independent of the number and mix of quota rights received. This results from the independence of the trading technologies $\hat{h}(\hat{s}_m)$ and $\hat{j}(\hat{s}_z)$.

¹⁶If good z is an exportable, the relevant break-even condition is $1 - t_x < \hat{B}_j$. Again, \hat{j}' satisfies $0 \leq \hat{j}' \leq 1$, so that commodity j will not be smuggled if its export tax rate is less than the black market premium.

foreign exchange rationing. In this model, a liberalization of the trade regime in the form of an export tax reduction may increase the black market premium on foreign exchange as a result of the decreased incentive to underinvoice exports, a source of supply to the black market.

The assumptions of the base model were relaxed to allow for capital flight, unbalanced trade, foreign exchange control, and quotas. It was found that increases in the exogenous demand for black foreign market exchange, perhaps for purposes of capital flight, increase export smuggling and the black market premium and reduce import smuggling. Surprisingly, an increase in the trade deficit has the same effect. Smuggling and the black market cannot be eliminated by the increased provision of legal foreign exchange if smuggling exists to avoid payment of trade taxes. The equivalence of foreign exchange controls and quotas with tariffs was established by demonstrating the irrelevance of the allocation of quota rights or foreign exchange licenses among firms possessing the same trade technology. Finally, the results were extended to the case of more than two goods where it was found that reducing an import tariff may cause the smuggling of an export to cease and initiate the smuggling of another import.

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