Pion distribution amplitude from Euclidean correlation functions: Universality and higher-twist effects

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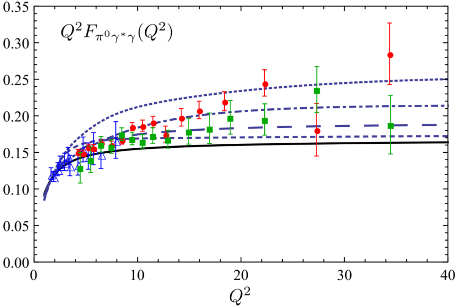
Definition of distribution amplitudes

- hard exclusive processes are sensitive to the distribution of the momentum within a Fock state at small transverse distances
- this information is contained in light-cone DAs; schematically:

$$\phi_\pi(u,\mu) \propto \int_{|\mathbf{k}_\perp|<\mu} d^2k_\perp \langle 0|\bar{q}(\bar{u},\mathbf{k}_\perp)q(u,-\mathbf{k}_\perp)|\pi\rangle$$

- quark and antiquark carry the momentum fraction u and $\bar{u} = 1 - u$, respectively
- complementary to PDFs, which do not discriminate between Fock states
- the DAs are related to experimental form factor data (e.g., on the $\pi^0\gamma^*\gamma$ transition) by perturbative QCD and light cone sum rules (LCSR)

The BaBar Puzzle



- plot taken from arXiv:1206.3968
- data from

  CLEO (1998, blue trianlges)
  BaBar (2009, red circles)
  Belle (2012, green squares)
- solid line: result obtained for the asymptotic pion DA $\phi(u) = 6u(1-u)$
- dashed lines: results for various DA models

- BaBar Puzzle: the continuous rising exhibited by the BaBar data seemed to contradict collinear factorization at intermediate momentum transfer
- the Belle data does not support such a conclusion anymore

  $\Rightarrow$ additional information from lattice QCD is highly valuable

Lattice calculation

- pion DAs are defined as pion-to-vacuum matrix elements with a quark-antiquark pair at lightlike separation connected by a Wilson line
- **problem:** on a Euclidean space-time one cannot realize nontrivial lightlike distances
- **traditional solution:** calculate Mellin moments of the DAs ($\hat{=}$ local derivative ops.)
  - higher moments $\rightarrow$ problems with renormalization (operator mixing)

- **new approach:** calculate correlation functions at spacelike distance

Option 1: use a nonlocal operator $\langle 0|\bar{q}(z)\Gamma[z,0]q(0)|\pi\rangle$

- Fourier transform $\rightarrow$ quasi-distribution $\rightarrow$ analysis via LaMET

  Ji arXiv:1305.1539, Zhang et al. arXiv:1702.00008
- direct analysis in position-space using pQCD Karpie et al. arXiv:1710.08288

Option 2: use two local operators $\langle 0|\bar{q}(z)\Gamma_1 q(z)\bar{q}(0)\Gamma_2 q(0)|\pi\rangle$

- direct analysis in position-space using pQCD

  Braun, Müller arXiv:0709.1348; Bali et al. arXiv:1709.04325

- all position space approaches require large hadron momenta

Matrix elements ↔ DAs

$$\mathbb{T}_{\mathrm{XY}}(p\cdot z, z^2) = \langle 0|J_X^\dagger(\tfrac{z}{2})J_Y(-\tfrac{z}{2})|\pi^0(p)\rangle$$

$$J_S = \bar{q}\,u\,,\quad J_P = \bar{q}\gamma_5 u\,,\quad J_V^\mu = \bar{q}\gamma^\mu u \equiv J_{V^\mu}\,,\quad J_A^\mu = \bar{q}\gamma^\mu\gamma_5 u \equiv J_{A^\mu}$$

$$\mathbb{T}_{\mathrm{SP}} = T_{\mathrm{SP}}$$

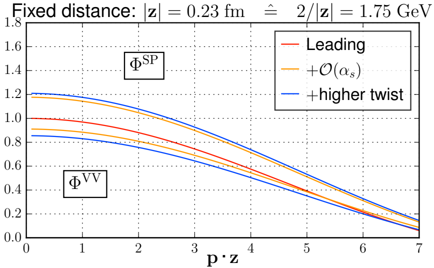
$$\mathbb{T}_{\mathrm{VV}}^{\mu\nu} = \frac{i\varepsilon^{\mu\nu\rho\sigma}p_\rho z_\sigma}{p\cdot z}T_{\mathrm{VV}}$$

$$\mathbb{T}_{\mathrm{VA}}^{\mu\nu} = \frac{p^\mu z^\nu + z^\mu p^\nu - g^{\mu\nu}p\cdot z}{p\cdot z}T_{\mathrm{VA}} + \frac{p^\mu z^\nu - z^\mu p^\nu}{p\cdot z}T_{\mathrm{VA}}^{(2)} + \frac{2z^\mu z^\nu - g^{\mu\nu}z^2}{z^2}T_{\mathrm{VA}}^{(3)} + \frac{2p^\mu p^\nu - g^{\mu\nu}p^2}{p^2}T_{\mathrm{VA}}^{(4)} + g^{\mu\nu}T_{\mathrm{VA}}^{(5)}$$

- similar for PS, AA, AV
- q is an auxiliary quark $q \neq u, d$, but $m_q = m_u = m_d$

Matrix elements $\leftrightarrow$ DAs

$$T_{XY}(p\cdot z, z^2) = F_\pi \frac{p\cdot z}{2\pi^2 z^4} \underbrace{\int_0^1 du\, e^{i(u-1/2)p\cdot z}\phi_\pi(u) + \mathcal{O}(\alpha_s) + \text{higher twist}}_{\equiv\, \Phi^{XY}(p\cdot z, z^2)}$$



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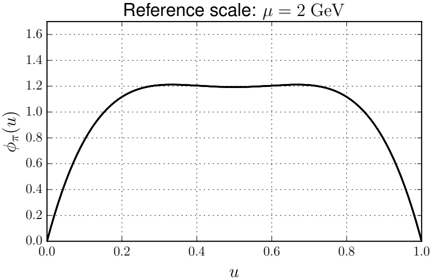

- twist 4 effects estimated using asymptotic shape for chiral-odd twist three DAs $\rightarrow$ one parameter $\delta_2^\pi = 0.17\,\text{GeV}^2$ (at $\mu = 2\,\text{GeV}$, QCD sum rule estimate)

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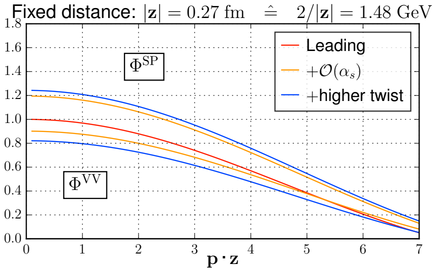

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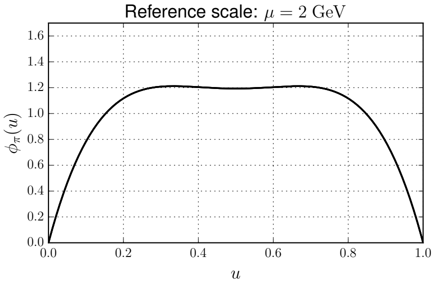

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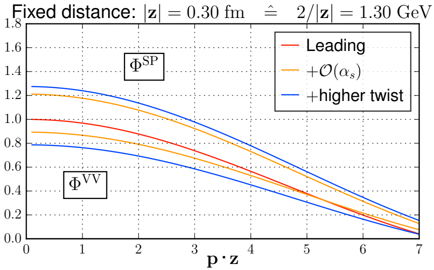

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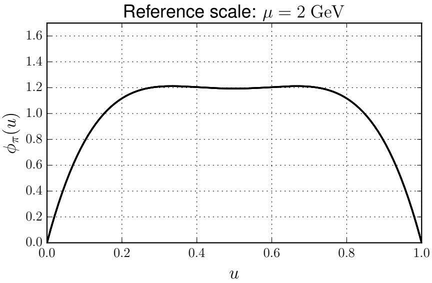

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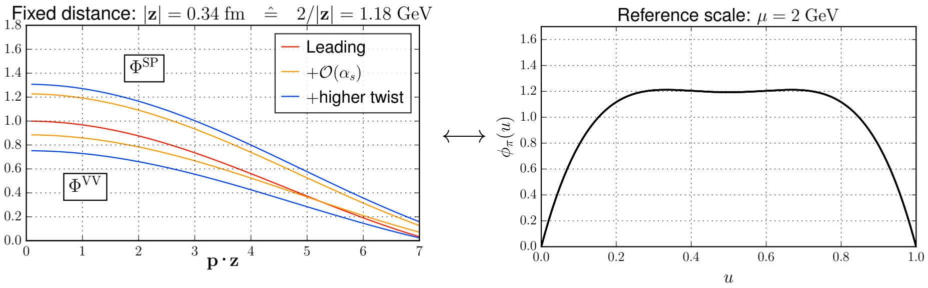

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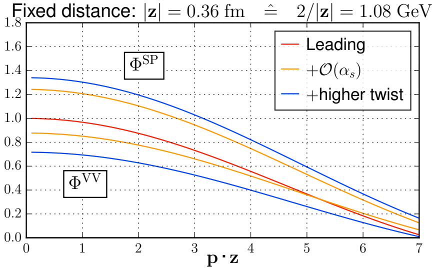

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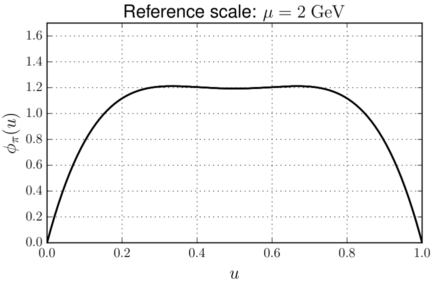

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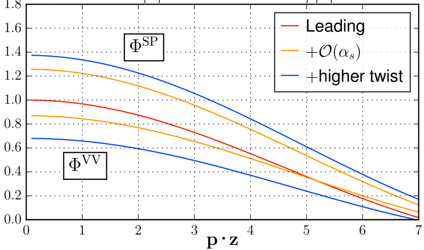
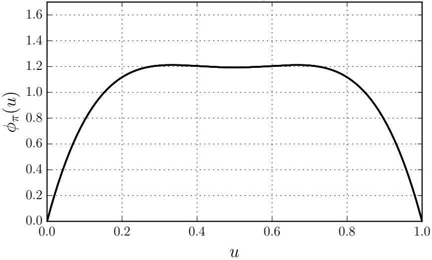
Fixed distance: $|\mathbf{z}| = 0.39$ fm $\hat{=}$ $2/|\mathbf{z}| = 1.01$ GeV



$\longleftrightarrow$

Reference scale: $\mu = 2$ GeV



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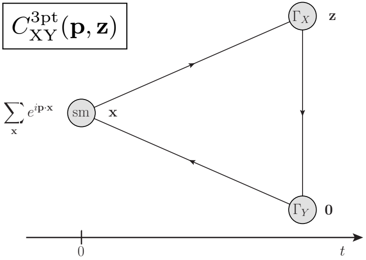
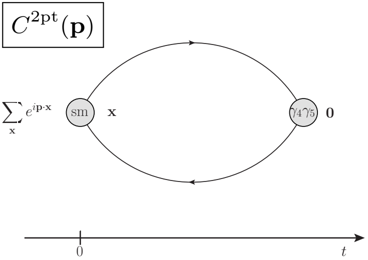
Obtaining the matrix elements from Lattice

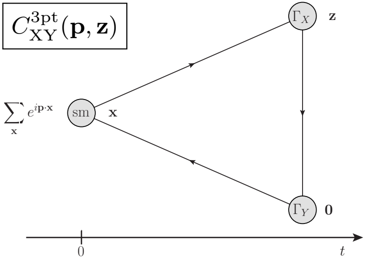


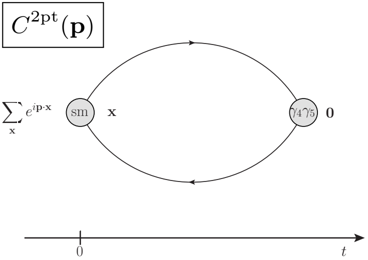


$$\frac{\mathbb{T}_{XY}(p\cdot z, z^2)}{F_\pi} = \frac{Z_X(\mu)Z_Y(\mu)}{Z_A} \frac{C^{3pt}_{XY}(\mathbf{p},\mathbf{z})e^{\frac{i}{2}\mathbf{p}\cdot\mathbf{z}}}{C^{2pt}(\mathbf{p})} E(\mathbf{p}) + \text{excited states}$$

- the Z_X is the renormalization factor for the respective current (nonperturbatively calculated in RI′-MOM $\rightarrow$ conversion to $\overline{\text{MS}}$ in 3-loop PT)
- we set both, the renormalization and the factorization scale to $\mu = 2/|\mathbf{z}|$
- phase factor shifts the currents to the symmetric position

Obtaining the matrix elements from Lattice





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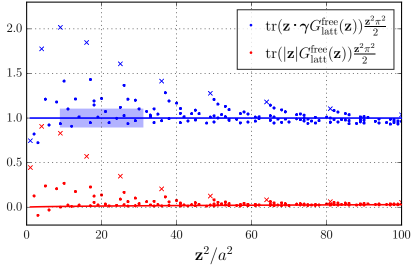
- smearing: momentum smearing — Bali et al. arXiv:1602.05525
  → improved overlap with hadrons at large momentum
- new: we use stochastic estimation
  → get a volume average at the cost of some stochastic noise
  → much smaller statistical error

Discretization effects of the free Wilson propagator

propagator comparison:

free Wilson vs. free continuum

- large effects in chiral even (blue, $\propto \not{z}$) and chiral odd (red, $\propto \mathbb{1}$) part
- in continuum: chiral odd part strongly suppressed
- **problem on lattice:** large artefacts from terms removing the doublers



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solution:

1. use observables, where the chiral odd part does not contribute at tree-level

$$\frac{1}{2}\left(T_{\mathrm{SP}} + T_{\mathrm{PS}}\right), \qquad \frac{1}{2}\left(T_{\mathrm{VA}} + T_{\mathrm{AV}}\right), \qquad \frac{1}{2}\left(T_{\mathrm{VV}} + T_{\mathrm{AA}}\right)$$

2. introduce correction factor for chiral even part
3. most important: ignore distances where the correction $> 10\%$ or $|\mathbf{z}| < 3a$

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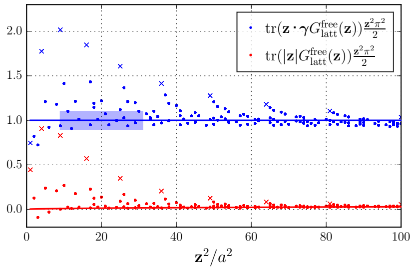
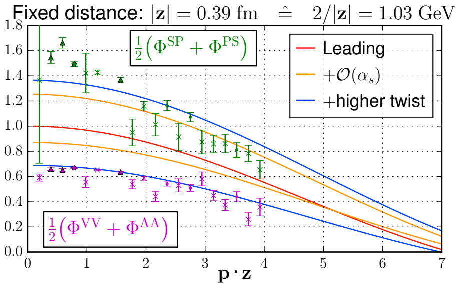


note:

1. upper limit of range determined by $\mu = 2/|\mathbf{z}| \geq 1\,\mathrm{GeV}$
   $\Rightarrow a \to a/2$ shifts the upper limit by a factor 4 to the right
2. discretization effects are strongest along the axes (crosses)
   $\to$ similar for Wilson-line operators?

Lattice simulation

- mass-degenerate $N_f = 2$ nonperturbatively improved Wilson (clover) fermions and Wilson gluon action
- $L^3 \times T = 32^3 \times 64$
- coupling parameter $\beta = 5.29 \widehat{=}$ lattice spacing $a \approx 0.071\,\text{fm} = (2.76\,\text{GeV})^{-1}$
- mass parameter $\kappa = 0.13632 \widehat{=}$ pion mass $m_\pi = 0.10675(59)/a \approx 295\,\text{MeV}$
- 12 momenta in different directions with $0.54\,\text{GeV} \leq |\mathbf{p}| \leq 2.03\,\text{GeV}$



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- general trend is correct
- nice demonstration of universality
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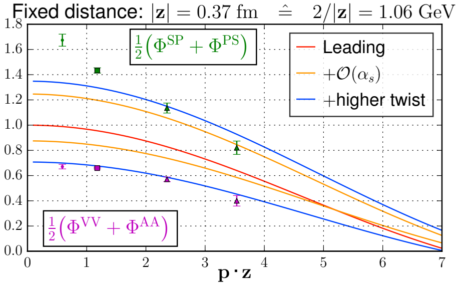

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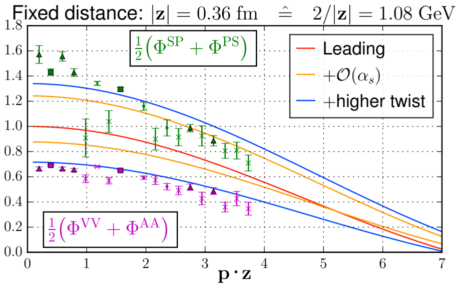

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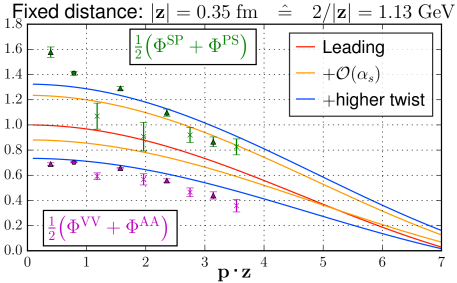

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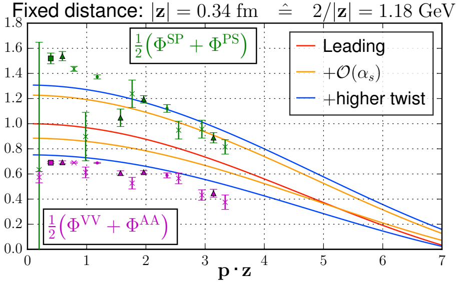

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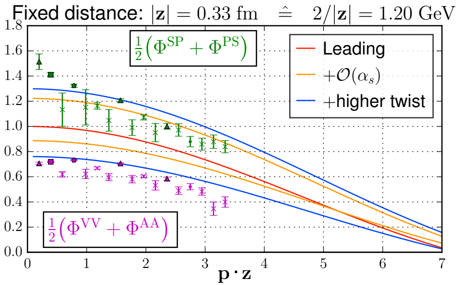

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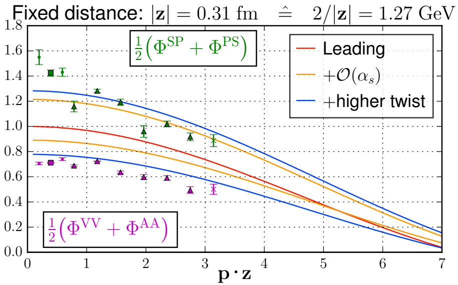

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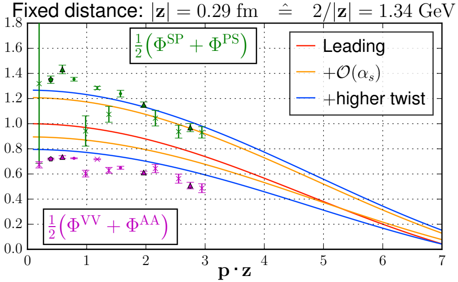

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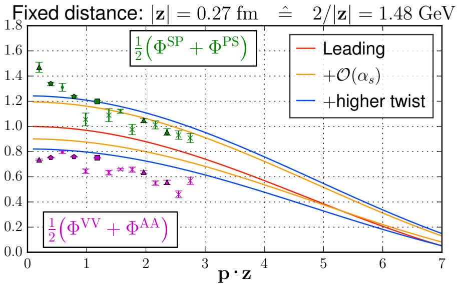

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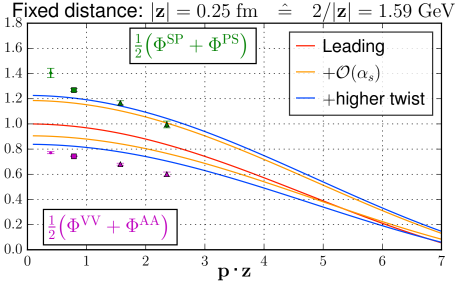

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Fit to data: parametrization of the DA

Expansion in orthogonal (Gegenbauer) polynomials

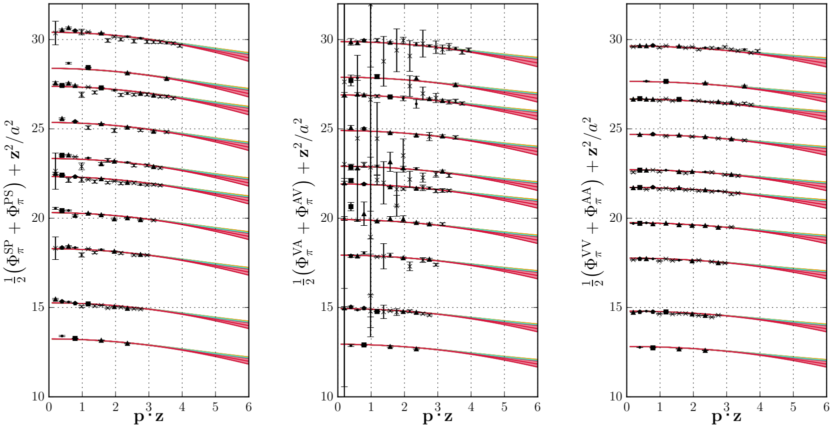
$$\phi_\pi(u,\mu) = 6u(1-u) \sum_{n=0,2,\dots}^{\infty} a_n^\pi(\mu) C_n^{3/2}(2u-1)$$

- $a_0^\pi = 1$ (normalization condition)
- the coefficients $a_n^\pi(\mu)$ do not mix under evolution at one-loop accuracy
- in the limit $\mu \to \infty$ higher coefficients vanish faster
- only even coefficients occur for the pion due to isospin symmetry
- we use a truncation at $n = 2$ (turquoise) and at $n = 4$ (red) as models for the DA at the scale $\mu = 2\,\text{GeV}$

As an alternative parametrization we will test (gold)

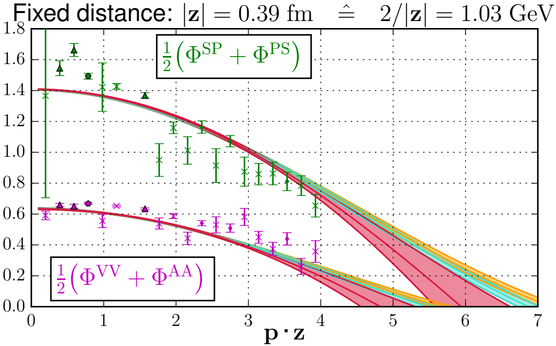
$$\phi_\pi(u, \mu = 2\,\text{GeV}) \propto \Big[u(1-u)\Big]^\alpha \text{ normalized to one}$$

Combined fit to all channels (Legacy Plot)



- two parameters: α, δ_2^π
- two parameters: a_2^π, δ_2^π
- three parameters: a_2^π, a_4^π, δ_2^π $\leftarrow$ yields unreasonable values for a_4^π

Combined fit to all channels



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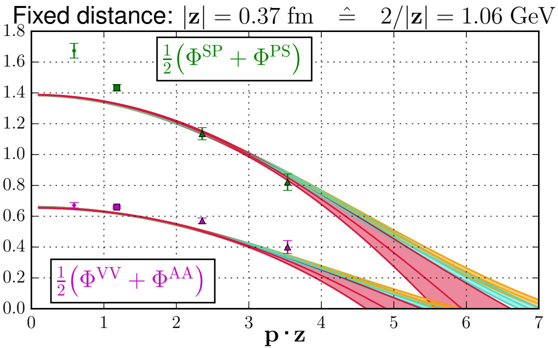

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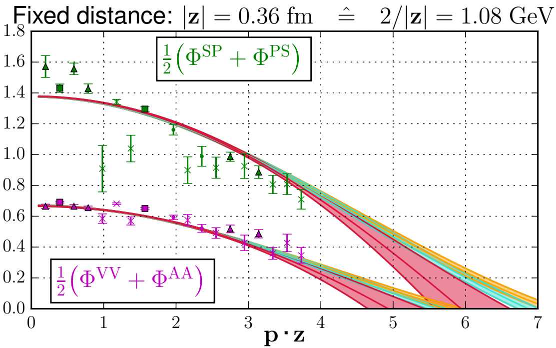

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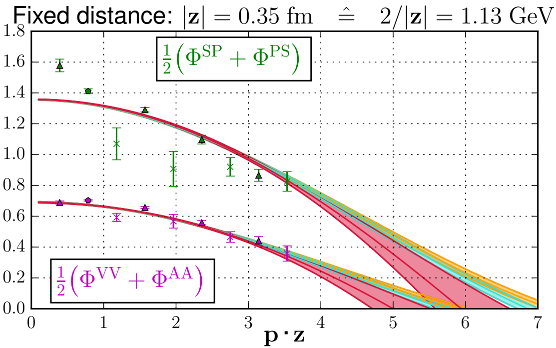

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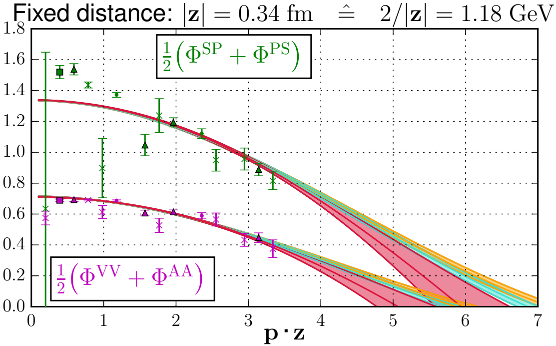

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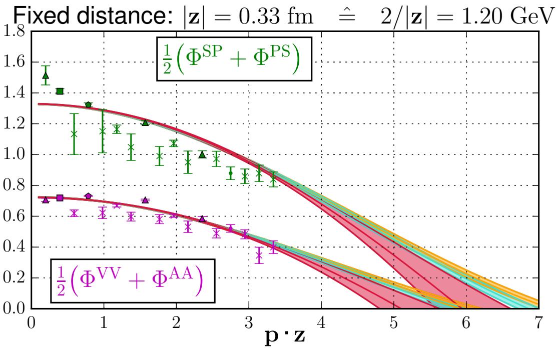

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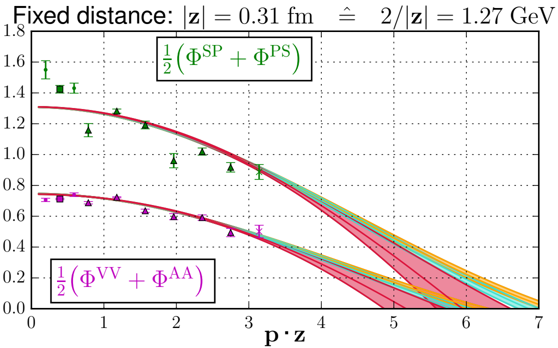

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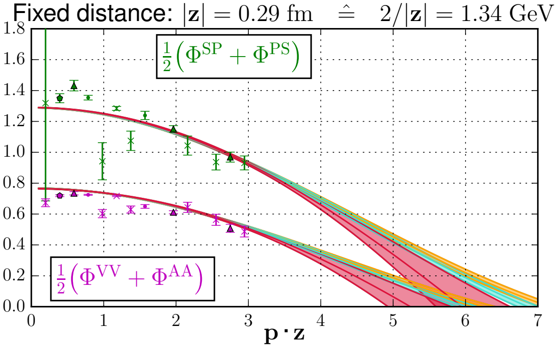

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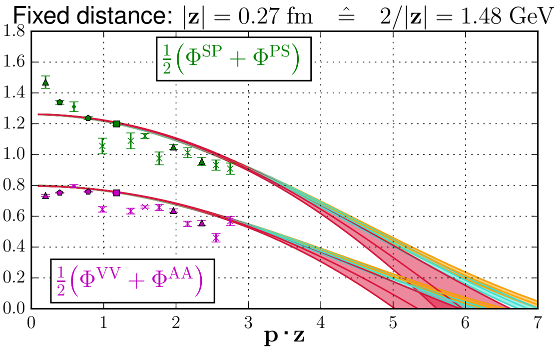

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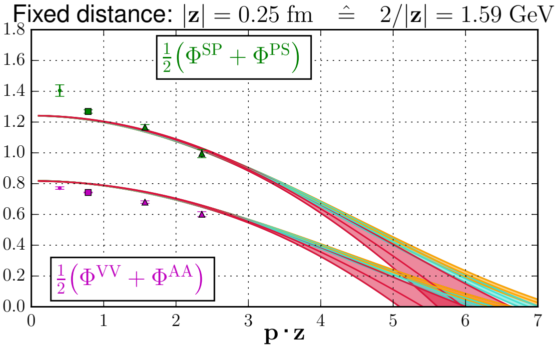

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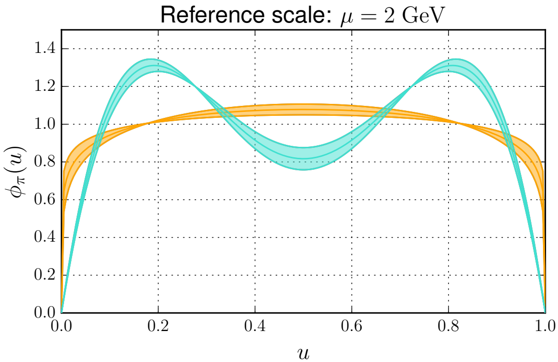
Result for DAs

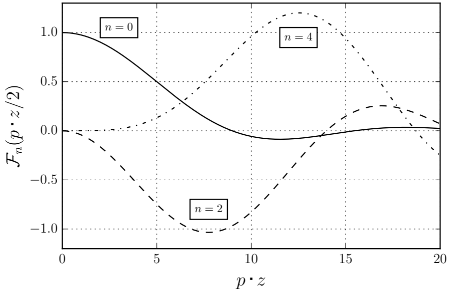


- errorbands show only the statistical error
- parameters: $\alpha = 0.13(5)$, $\delta_2^\pi = 0.223(4)$ GeV^2 $a_2^\pi = 0.30(3)$, $\delta_2^\pi = 0.223(4)$ GeV^2
- both agree perfectly well with our data: **Why?**
- only relevant information from DA for our data points is a_2^π and $a_2^\pi = 0.31(3)$
- **Disclaimer:** current systematic uncertainty for a_2^π, δ_2^π is at least $\approx 50\%$ (fit range variation, estimate for two-loop correction)

Whats the problem with a_4^π?

$$\phi_\pi(u,\mu) = 6u(1-u) \sum_{n=0,2,\ldots}^{\infty} a_n^\pi(\mu) C_n^{3/2}(2u-1)$$

$$\Rightarrow \quad \Phi^{\mathrm{XY}} = \sum_{n=0,2,\ldots}^{\infty} a_n^\pi(\mu)\mathcal{F}_n(p\cdot z/2) + \mathcal{O}(\alpha_s) + \text{higher twist}$$



Expansion in conformal partial waves $\mathcal{F}_n$

- one needs $p\cdot z \gtrsim 5$ to constrain a_4^π to reasonable values
- to discriminate between DAs on last slide: $p\cdot z \gtrsim 8$?

Summary

- we have analysed Euclidean correlation functions with two *local* currents
- global fit to multiple channels yields qualitatively reasonable results (universality)
- first determination of HT normalization δ_2^π from lattice QCD (in the ballpark of QCD sum rule estimates)
- statistical accuracy very good for a_2^π and δ_2^π

BUT:

- systematic uncertainty for a_2^π and δ_2^π is very large (discretization effects, two-loop perturbative correction not taken into account)
- with current data no determination of a_4^π possible

Next steps:

- goto smaller lattice spacings ($a \approx 0.04\,\text{fm}$ would be nice)
- perturbative two-loop calculation for coefficient functions
- to be sensitive to a_4^π: goto larger momenta ($|\mathbf{p}| > 3\,\text{GeV}$ would be nice)